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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF EDUCATION

DEPARTMENT OF SECONDARY EDUCATION

EDMONTON, ALBERTA

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THE RELATIONSHIP BETWEEN COGNITIVE STYLES AND MATHEMATICS ACHIEVEMENT IN TWO TYPES OF MATHEMATICS LABORATORIES

by

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ABSTRACT

This study investigated an aptitude-treatment interaction. Two cognitive aptitude variables, Skemp's reflective intelligence and Kagan's conceptual tempo, were studied in relationship to two distinct types of mathematics laboratories.

A mathematics laboratory is a method of instruction which requires the student to actively manipulate concrete materials either individually or in small groups in order to discover a mathematical concept. The two mathematics laboratory programs comprised of eighteen lessons were developed from a unit on probability. The sample consisted of four classrooms of grade ten non-academic students. The students were grouped into pairs according to a friendship criteria and then randomly assigned to either of the two treatments. The two treatments, the directed and non-directed laboratories had parallel activities but differed in instructional technique. The major difference between the programs was that the directed laboratory had a set of questions (advance organizers) placed at the beginning of the laboratory lessons.

A 30 question multiple-choice achievement exam was administered at the completion of the three week program. The achievement test was used as the dependent variable in a multiple regression interaction analysis. Measures from Skemp's test of reflective intelligence and from Kagan's Matching Familiar Figures Test were used as independent variables. Reflective intelligence is the ability of the mind to become aware of concepts and to be able to manipulate these concepts. Skemp has shown that reflective intelligence is a good variable for predicting mathematics achievement. Kagan's conceptual tempo variable measures the

speed at which a subject replies to a question which has several response uncertainties. He has described two types of individuals - the impulsive and the reflective thinker. The impulsive individual is characterized as a person who makes fast unconsidered responses and many errors while the reflective individual is described as a person who considered alternative solutions to a problem before responding to questions and, therefore, makes fewer errors than the impulsive individual.

The results of the study showed that there was no interaction between Skemp's variable and the achievement score in the two types of laboratories and there was no interaction between Kagan's variable and the achievement scores. Since there was no interaction, the main effects were tested. It was found that Skemp's reflective intelligence was a good predictor of achievement for the sample and that reflective intelligence is a separate variable from that of general intelligence as measured by the Lorge-Thorndike I.Q. test. No significant difference was found between the directed and non-directed laboratory. The reason for lack of aptitude-treatment interaction between the variables used in the study could be the results of an insignificant instructional difference between the two laboratory settings. Also there was no significant difference between the achievement scores of the impulsive and reflective students. Since the sample used in the study was low achieving students, it is possible that the median split used to divide the student into reflective and impulsive students was invalid for the sample.

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THE NATURE OF THE PROBLEM

I. Introduction

In the last decade, educational research and development in the field of mathematics has focused mainly on the average and above average college-bound student. However, the recent thrust in educational research has changed from the college-bound student to the low achieving, non-academic student. Responding to this new interest, the National Council of Teachers of Mathematics (1970) produced some badly needed materials for grades five to eight (Chandler, 1970). These materials were designed to motivate the low achievers through activity learning by stressing the following principles:

1. Activity: Mathematics must not be considered a spectator sport where the students watch rather than participate (Polya, 1966). The students should be given a chance to interact with each other as well as with the environment.
2. Individuality: According to Mitzel (1970) each student should be given opportunity to work at his own rate, to work at times convenient to him, to have a choice of instructional strategies, to have a chance to develop missing skills and to start at his level of comprehension.
3. Success: Since the low achiever has been unsuccessful in earlier attempts to understand mathematics, success is very important (it is assumed that success breeds success).

By breaking down the components of the lesson into very small tasks, success is more likely.

4. Meaning: Mathematics should no longer be taught as memorizing of a formula or analogue but rather as understanding through manipulation of materials and interaction with the environment.

5. Novelty: Tasks are varied so that the learner is confronted with problems which are novel and motivating to him.

This report discusses a special type of activity learning, the mathematics laboratory. A mathematics laboratory is a method of instruction in which the learner physically manipulates concrete materials in order to actively participate in the development of a mathematical concept. In a review of recent research on "activity" learning in mathematics, Kieren (1969) suggested that there are two main areas of research. The first, the "discovery" approach is a method in which the student actively participates in forming mathematical concepts although he does not use any physical devices to aid learning. The second, the "manipulative" approach has the learner manipulate physical materials to help discover mathematical concepts. Although there is no lack of theoretical discussion on the usefulness of manipulative learning in mathematics, Kieren has pointed out that the actual quality of research done on manipulative activities is questionable since most of the research has been in the form of pilot studies.

It should be noted that the mathematics laboratory requires the

student to actively "discover" the mathematical concept by himself. Perhaps, George Polya (1962) stressed this notion of heuristic teaching more elegantly when he said:

> " It has been said by many people in many ways that learning should be active; not merely passive or receptive: merely by reading books or listening to lectures or looking at moving pictures without adding some action of your own mind you can hardly learn anything and certainly you cannot learn much.
>
> There is another form expressed (and closely related) opinion: <u>The best way to learn anything is to discover it yourself</u>. Lichtenberg (an eighteenth century German physicist, better known as a writer of <u>Aphorisms</u>) adds an interesting point: <u>What you have been obliged to discover by yourself leaves a path in your mind which you can use again when the need arises</u>. Less colourful is the following statement, but it may be more widely applicable: <u>For efficient learning, the learner should discover by himself as a large, a fraction of the material to be learnt as feasible under the circumstances</u>.
>
> This is the <u>principle of activity learning</u> (Arbeitsprinzip). It is a very old principle: it underlines the idea of "Socratic method". (1962,P . 3-4).

Higgins (1971) has suggested that the word heuristic has "an infinitely richer meaning than simply 'discovery' " (1971, P. 487). It encompasses the study of the methods and rules of discovery and invention. He has said that heuristic teaching is very closely related to problem solving and that one can teach heuristically by using questions built from a logical strategy. These questions can lead the student to "uncover" or "discover" the mathematical idea in a problem-solving setting. The mathematics laboratories developed in the study attempted to incorporate this principle of heuristic teaching.

The sample in this study was composed of low achieving grade ten mathematics students. Activity oriented materials were used to teach the students new mathematical concepts in a laboratory setting. These materials de-

veloped involved the principles of activity, individuality, success, meaning and novelty as advocated by the National Council of Teachers of Mathematics (1970).

II. The Problem

There are many student characteristics which could be discussed in the selection of instructional tactics. Although many theorists, most notably Cronbach, have argued persuasively for the importance of individual differences in learning, research in this areas has been very scanty. For example, the students prior knowledge in the area of instruction is an important factor in the selection of instructional tactics. Successful expository or discovery teaching is contingent on the previous knowledge of the student. This would suggest that minimal prerequisites of understanding are necessary before effective instruction can take place.

The learner's developmental level is another important factor affecting the choice of instructional tactics. Piaget, has observed that an individual can learn only those concepts which he is mentally ready for. The learning of these concepts depends on the child's development of operational structures which begin at the sensory-motor level and develops to the formal operational level. Piaget has also observed that these levels of development must follow in order although the age of maturity may differ greatly amongst individuals (Adler, 1963). Hence, Piaget would view the child's level of cognitive development as a major criterion in selecting the mode of instruction.

An additional variable to consider would be the learner's cognitive style. Unfortunately not much research has been done in this area. As

Shulman (1970) has pointed out this type of variable "combines intuitive feast with empirical famine. The convincing data on learning relevant individual differences in cognitive style are rare indeed. On promising stylistic variable is Kagan's Conceptual tempo dimension" (1970, P. 65). Conceptual tempo is defined as the speed with which a person responds to a question which has response uncertainty. Another cognitive variable which could be studied is Skemp's variable of reflective intelligence. According to Skemp (1958) this variable measures the mathematical aptitude of the student.

It is the purpose of this report to discuss Kagan's variable of reflective and impulsive personalities and Skemp's variable of reflective intelligence in relation to two types of mathematics laboratories. A mathematical laboratory can be defined as a method of instruction in which a learner discovers a mathematical principle for himself, at his own rate through actively participating in manipulation of concrete materials and socially participating in a group of two or three students. Two distinct types of mathematics laboratories were defined and developed to find out if one method of presenting material was superior to another method for some particular type of student. Basically the two laboratories, directed and non-directed laboratories, differed in the amount of time spent in free or unstructured play. A measure of achievement from these laboratory lessons was used to predict which type of learner benefited the most from the directed or non-directed laboratory setting.

The study attempted to answer questions related to the major question of what type of learner benefits most from a particular type of mathematics laboratory setting. Answers were sought for the following questions:

1. A. Do impulsive students in directed laboratories achieve better than reflective students? And, do reflective students generally achieve better in the non-directed laboratory? In general, is there any interaction between the two types of mathematics laboratories and the impulsive and reflective variable in achievement?

   B. Are reflective students generally higher achievers than impulsive students?

2. A. Do students with high scores in Skemp's test achieve better in the non-directed laboratory? Or, do the students with low scores in Skemp's test achieve better in the directed mathematics laboratory?

   B. Do students with high reflective intelligence receive high achievement scores and likewise do students who have low reflective intelligence receive low achievement scores?

III. Need for the Study

In a brief presented to the National Council of Teachers of Mathematics, Becker (1970) pointed out the need to develop a mathematical theory which would answer the question of how mathematics can be effectively taught and effectively learned. It was hoped that this theory would effectively unify works from a variety of disciplines, in particular, mathematics education and behavioral psychology. Individualized instruction would provide one basic aspect of the framework around which the new theory could be developed. According to Mitzel (1970):

> "The general goal of society should be a genuine Adaptive Education, by which we mean the tailoring of subject-matter and presentations to fit the special requirements and capabilities of each learner. The ideal is that no learner

should stop short of his ultimate achievement in the area of content because of idiosyncratic difficulties in his study strategy ... because the nature and extent of human variability is so overwhelming we are forced to conclude that adaptive education is almost completely dependent upon the individualization of instruction". (1970, P. 14).

It should be noted that individualized instruction does not necessarily imply individual instruction. The latter connotes that the learner is in isolation while the former suggests that some variable or variables have been taken into account in building the program. The following study takes into account two possible variables: Skemp's variable of reflective intelligence and Kagan's variable of reflective and impulsive personalities.

There are many different programs of individualized instruction in use in the schools. For example, mathematics laboratories, Programs for Learning in Accordance with Needs (PLAN), Individually Prescribed Instruction (IPI), and Computer Assisted Instruction (CAI) were developed to fill a need for individualized instruction. Unfortunately very little research has been done with different aptitude variables in relation to achievement scores. Becker (1970) makes a strong case for studying aptitude-treatment interaction theories: As he stated:

"If such a theory were formulated, it might well include statements about individual differences, about different methods of teaching and about the relationship between these. There has been a great deal of research done which is related to these things. But, by and large, these studies are concerned with the general question, 'Can we teach as well by method X as by method Y?'. That is, much of the research is concerned only with the existence or non-existence of significant differences in main effects (e.g. that one method of teaching is better than another, on the average). I do not assert that these studies are not valuable, but I do say there is another important question to be asked, one that is more pertinent, namely 'For which student is a particular method of instruction most effective?' (1970, P. 22).

Kieren (1969) and Vance (1969) also suggested that studies involving factors affecting a students ability to learn mathematics should be piloted especially with regard to the mathematics laboratory.

In a discussion about low achievers Johnson and Rising (1967) have pointed out that it is not just the slow learner with below average academic ability who is a poor achiever in mathematics, but there are many other types of students such as the under achiever or the rejected learner. Since the low achievers form a heterogeneous group one technique of instruction might not be the best method of instruction for all. For this reason a number of cognitive variables were studied. The aim of studying these variables was to ascertain which type of student benefited most from a particular type of mathematics laboratory.

IV. Definitions

Mathematics Laboratory: A method of mathematical instruction which requires the learner to actively manipulate concrete materials either individually or in small groups.

Directed Laboratory: A mathematics laboratory in which each lesson was introduced with advance organizers. Following the advance organizers, the student was required to either play a game or solve a major problem using concrete materials. Finally the mathematical concepts were clarified and reinforced through questions.

Non-Directed Laboratory: A mathematics laboratory in which the students were introduced to a game or a major problem followed by questions which clarified and reinforced the mathematical concepts.

Advance Organizers: A set of questions usually in the form of a tree diagram which is used to introduce the game or problem in the directed laboratory.

Low Achievers: A group of students with one common trait: that is, they had all failed the previous years work in mathematics. According to Johnson and Rising (1967) the low achievers are not a homogeneous group. They include such students as the slow learner, under-achiever, reluctant learner, disadvantaged learner, the culturally deprived learner and so forth.

Impulsive Thinker: An individual who makes fast unconsidered responses to a problem and, therefore, makes many errors.

Reflective Thinker: An individual who considers alternative solutions to a problem and, therefore delays his response thereby making fewer errors.

Reflective Intelligence: The ability of the mind to become aware of concepts and the ability to manipulate these concepts. Skemp has shown that reflective intelligence is separate from general intelligence and that is it useful in predicting mathematics achievement.

Verbal Battery: A subtest of verbal items only from Lorge-Thorndike I.Q. Test. It is a good index of scholastic aptitude.

Non-Verbal Battery: A subtest of either pictorial or numerical items from Lorge Thorndike I.Q. Test. It gives an estimate of

scholastic aptitude which has not been influenced by lack of reading ability.

V. Experimental Setting

The study involved four different Edmonton mathematics classes - two at Victoria Composite High School, one at Queen Elizabeth Composite High School and one at Strathcona Composite High School. The students in these classes were all low achievers, that is, they had all failed the previous years work. Two parallel forms of the mathematics laboratory, the directed and non-directed laboratory were developed in conjunction with a unit in probability from the Grade 10, Math 15 curriculum. The students were divided into groups of two and then randomly assigned to either the directed or non-directed laboratory.
The students were not aware of the experimental nature of their assignments.

VI. Delimitations of the Study

1. The study involved four non-matriculation Grade 10 classes.
2. The study was designed to be about 1,000-1,600 minutes of instruction over a three to four week period.
3. Only one area of mathematics content was sampled - probability and related counting problems.
4. The study took place in three Edmonton high schools which operated on the semester system and, therefore, had 80 minutes of lesson time each day.

VII. Outline of the Report

The present chapter has introduced the problem - what type of mathematics laboratory is most beneficial to a particular type of student. The developmental principles behind the mathematics laboratories are discussed in Chapter II. The first part is a review of the mathematic learning theories which have to do with manipulation of concrete materials. In particular, Piaget, Bruner, Skemp and Dienes' theories of concept development are discussed. The second part of Chapter II deals with the developmental portion of the study. A description is given of the rationale of the mathematics laboratory, the distinction between the directed and non-directed laboratory, and a brief outline of the materials that were developed. Chapter III is divided into two parts: a discussion about Kagan's variable of impulsivity and reflectivity and Skemp's variable of reflective intelligence. Each part describes the background of the test, the instrument used, the research findings related to the study and the theoretical framework of the study. Chapter IV gives a description of the design of the study including the hypothesis. Chapter V gives the statistical results of the research portion of the study and Chapter VI discusses the conclusion that can be drawn from the study and suggests further research which can be done in this area.

CHAPTER II

THE DEVELOPMENT OF THE MATHEMATICS LABORATORIES

The developmental problem of the study is discussed in this chapter. The first portion of the chapter deals with a review of the literature related to the theory of manipulative learning and to the recent research done in the area of manipulative instruction. Piaget's, Bruner's, Skemp's and Dienes' theories of mathematics learning are discussed with special emphasis in the area of concept development. The second portion of the chapter is a discussion of the development of the two types of mathematics laboratories used in the study.

I. Learning Theories

The following review will look at the work of several well known cognitive psychologists and their theories in the development of concepts. Their theories were discussed either since they advocate activity learning or their theories were used in the development of the rationale for the mathematics laboratory. Piaget's, Bruner's and Dienes' theories were reviewed since they advocate activity learning. Unfortunately, their theories are mainly developed around observations and research done in the elementary grades. Skemp's theory, on the other hand, is centered around high school research, his reflective intelligence occurring after activity learning as a result of abstraction and generalization of concrete concepts.

A. Piaget

Piaget's studies differentiated two problems — the problem of development and the problem of learning. The developmental problem discusses the spontaneous growth of intelligence from infancy to adulthood while the learning problem is limited to the development of a single cognitive structure at any moment in time. Piaget has observed that intellectual development begins with physical actions upon the environment. Later, these physical experiences along with logical (or mathematical) experiences gradually help the child construct a stable body of information about the physical world. Piaget's research suggests that new cognitive structures are developed by learning general principles which subsume less general principles. These new principles cause conflict with the established schemas. A schema is a cognitive structure common to all those acts that an individual considers to be equivalent. In other words, an intellectual act is a process of trying to maintain equilibrium between assimilation schemas and information which has been accommodated. The assimulation process takes place whenever the subject acts in a new situation as he has acted in similar situations in the past. Accommodation, on the other hand, is the adoption of new concepts into the subject's repertoire of responses. It is the teacher's responsibility to continually introduce mild conflicts within a problem setting to ensure intellectual development (Adler, 1963).

In the development of operational structures, Piaget has distinguished four distinct stages: sensory-motor (one to eighteen months), preoperational (two to seven years), concrete operations (seven to eleven

or twelve years), and formal operations (twelve years and older) (De Cecco, 1968). The last two periods of Piaget's developmental theory contain important principles for developing a mathematics laboratory. In the concrete operations stage, basic concepts of closure, reversibility, associativity, and identity are acquired and organized into stable structures. These concepts are still limited to the immediate present unlike the later stage of formal operations where the child is capable of abstract thought.

The development of a child's operational structures depends on four factors: maturation, experience, social transmission and equilibration (Harrison, 1968). Piaget has observed that maturation or the age of readiness to operate at a different developmental level can differ greatly but that the order in which the stages take place is invariant. He also observed that initial experience should be at the concrete level to enable later experiences to be internalized at the logical (mathematical) level. Social transmission, the third factor, is the linguist or educational process the child encounters. For example, Piaget has observed that formal thinking corresponds to the age at which society expects the child to begin assuming adult roles. It is not the onset of puberty but the pressures of society which causes this change. Equilibration, the fourth factor, is the process of compensating for external disturbances to reach a state of equilibrium. Piaget has been reported as saying that even adults can learn better by <u>doing</u> than by being told about things (Harrison, 1968). He has also said that concrete experiences should be available whenever possible.

What is a concrete experience? A concrete experience is concrete relative to a person's past experience. For example, a child might understand addition as the process of adding two beads and three beads, or, a school age child might think of adding as 2 + 3 = 5, whereas, an adult might think of adding as X + Y = Z. This is Piaget's notion of vertical décalage. He defines décalage as the formal similarity between the structure of thinking at one level of development and at a higher level of functioning (Harrison, 1968). Flavell (1963) suggested that one implication of Piaget's theory is that a child will first learn at the concrete level. Later, in accordance with the hierarchy of learning, he will pass from concrete manipulation to pictorial representation, from a pictorial representation to symbolism, and from symbolism to mental operations.

B. Bruner

In the book, A Study of Thinking, Bruner (1956), stated that concepts are learned through adaption and modification of existing concepts rather than the acquisition of new concepts. "Virtually all cognitive activity involves and is dependent on the process of categorizing" (1956, P. 246). The categorization or classification process starts with a number of existing objects, each of which the subject must encounter and make a tentative prediction about. The prediction is then confirmed or invalidated. If it is found that the prediction is correct, the new information is then attributed to the object. These concept strategies tend to maximize information, reduce mental strain on the individual's capacity and help regulate the individual's behavior according to limita-

tions set by time or society. This method of concept attainment implies the use of a spiral curriculum in the school. That is, basic concepts of mathematics, science, or the arts should be introduced in concrete ways at an early level to enable students later to develop new higher level concepts on the former more basic concepts.

When Bruner (1966) says "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" (1966(a), P. 33), the assumption is that the subject is translated into the child's level of comprehension. This statement appears to be a contradiction to Piaget's development theory. Piaget would say that Bruner has overlooked the biological character of concept development. By examining Bruner's idea more closely this contradiction can be erased. He says that any subject matter can be developed at a concrete level and later it can be developed at a more abstract level. It should be further noted that Piaget is an observer and not a teacher of behavior. What he observes is the way things have been taught, and not, ways it might be possible to learn if teaching methods were changed (Harrison, 1968).

Bruner defined three ways to represent experience (Bruner, 1966(b)). The first, enactive representation, is representation of past events through appropriate motor responses. Next in the heirarchy is ikonic representation or the organization of images. The final representation is through arbitrary symbols. Mathematical concepts can be taught on any of these levels of representation. The best technique for learning is to advance from one representation level to the next although in the later stages of development some students are able to by-pass the first two steps. In secondary school, we often by-pass too many of these steps, leaving the

student nothing to fall back on if he fails to understand. Mathematics laboratories have been developed to help provide for more concrete instructional representation. Concrete materials are used for the assimulation of new abstract ideas (enactive representation). These ideas are then formulated into generalities (ikonic representation), and finally a notation system is used to describe the construction (symbolic representation). This final step is free of concrete manipulation and images.

Bruner (1966(b)) has suggested that all stages of learning require some manipulative activities with concrete materials. The question is how to strike a balance between the two extremes-- abstraction and concreteness. Inhelder (1962) agrees with Bruner that development of concepts is based on the activity of the child. He also felt that the child must have active manipulative experiences in a self explored situation.

C. Dienes

Dienes (1965) defines mathematics as a structure of relationships between different concepts which are connected with numbers (pure mathematics), and applications arising from the real world (applied mathematics). He feels that the complicated symbol system used by mathematicians is nothing more than a method of communication. Learning mathematics, therefore, is the apprehension of relationships and symbols. When applying this definition to teaching, emphasis should be placed on the process rather than on the correct responses. At the moment, most schools work on a reward-punishment basis for correct responses. The learning done

in this type of school can be described by the stimulus-response (S-R) theory of concept learning. Both Dienes (1963) and Scandura (1968) disagree with the S-R theory and have adopted a theory similar to Bruner's theory of categorization. Scandura's set-function language (SFL) and Dienes' predicate-subject theory suggest that more emphasis should be placed on structure rather than on content in teaching mathematics.

Dienes (1963) has developed three principles for mathematics learning: the Principle of Dynamics, the Mathematical Variability Principle and the Perceptual Variability Principle (Harrison, 1968). The Dynamic Principle suggests three basic stages in the development of a mathematical concept. The first is an unconscious or play stage. In this stage activities should be free and undirected, for example, when a baby plays with sounds before trying to communicate. The second stage is a slow realization that there is a path along which our experiences can be directed. In this period divergent thinking takes place. Suddenly, "insight" results in the solution to the problem. This state is the beginning of mathematical experience. The final stage involves the practice of the newly learned concept.

The second principle is the Mathematical Variability Principle. This principle implies that as many experiences involving a large number of variables as possible should be available when learning mathematics. The third principle, the Perceptual Variability principle implies that it is important to use many different materials to demonstrate the same concept. This principle, also known as the multiple embodiment principle, suggests that if students become too equipment oriented, transfer of the

concept is limited, although the one skill may be learned better. For example, the Cuisenaire rods were used with students in Trinidad. The students were shown to do better in calculations than the conventional classes. However, they were not able to transfer ideas to problem-solving situations as well as those in the conventional class, because they had become too instrument oriented (Vance, 1969).

E. Skemp

Skemp (1958) formulated a three-part theory of mathematics learning in which he tried to explain what a child needs besides general intelligence to succeed in mathematics. His theory in essence was:

1. that mathematical concepts can be learned efficiently.
2. that there is a schematic method of learning mathematics and,
3. that mathematics learning is dependent upon reflective intelligence.

Mathematics is probably the most interdependent and hierarchial of any structure of knowledge currently being taught. Concepts can be taught efficiently only if the relationship between previously learned concepts is fully understood. For example, one of the simpler concepts is the class-concept. This concept defines a set of properties which characterizes a particular class of objects. A child has begun to understand the class concept "animal" when he realizes that a collie is a dog and that all dogs are animals (Harrison, 1968).. This relationship between sets and subjects is very important to mathematics. Relations such as "less than" and "more than" and operations such as addition and subtraction are some other examples of mathematical concepts.

According to Skemp, there are two levels of concept attainment. Primary concepts are developed from direct sensory-motor experience; that is, experience gained from physical objects or action on physical objects. Secondary concepts are more abstract in nature, since they are from primary concepts. According to Skemp, Dienes' theory of mutiple embodiment involves only primary concepts and not secondary concepts since the child uses concrete objects to demonstrate the idea rather than mentally reflecting upon ideas already known to him (Harrison, 1968).

Schematic learning is defined as the way previous learning influences (makes possible) subsequent learning. The basic learning schemata - speaking, reading and writing - are one's own language. Unfortunately, a teacher will often teach a new concept at a language level unavailable to the child. Then the child must rote learn the new concept. Since rote learning does not require previous schemata to learn the new concept, the child is not able to completely understand the new idea and subsequently the child can not build any new concepts on the former one.

Skemp says that learning new mathematical concepts involves two processes. The first is the invention of new concepts by mentally combining old ones. (This theory is similar to Bruner's theory of categorization) The second is learning or practicing these combinations. Unfortunately, a child is often taught to memorize a required concept without first seeing the invention of the new concept. This type of learning is exemplified in the S-R (stimulus - response) theory where the punishment - reward system is used to teach a new concept.

Educational recommendations from Skemp's theory of concept formations are:

1. that students be provided with experiences needed to build operational thinking structures and,
2. that explanations given to the student be in terms that can be assimulated by the student (Harrison, 1968).

Reflective intelligence is a system which is able to turn inward on itself. Skemp (1958) defined reflective intelligence as the ability to rearrange, modify or choose from one's previous experience. It is this type of thinking that is required for a flexible creative approach to new problems. (In this case, problems are defined as questions which cannot be answered by routine applications of already known methods). According to this theory, a child who is performing poorly in mathematics has either not formed the necessary concepts and/or can not reflect on the concepts.

Piaget (1952) observed that a child can give correct responses to a problem without being able to explain the method he used to obtain the answer. This experiment can be explained by Skemp's theory of reflective intelligence since it demonstrates the dependence of simple arithmetic on sensory-motor intelligence and not on reflective intelligence. Sensory-motor intelligence is defined as the perception of relationships between objects and groups of objects presented to the senses, hence sensory-intelligence, and ones actions with these objects, hence motor-intelligence. Reflective intelligence, on the other hand, is a second order mental system which can perceive relationships among and act upon the concepts and operations of the sensory-motor system.

Harrison (1967) hypothesized that the measure of reflective intelligence increases with age. This helped to substantiate that Skemp's concept of reflective intelligence is closely related to Piaget's development theory. Sensory-motor intelligence should not be confused with the sensory-motor period of Piaget's developmental theory. Sensory-motor intelligence is actually at the concrete operational level since it is the perception of relationships between objects or groups of objects at the physical level (Harrison, 1968). It is reflective intelligence which is required in the learning of mathematics at the abstract level.

F. Recent Research

In a literature review of research in mathematical teaching methods, Kieren (1969) and Vance (1969) noted that research on manipulative learning has become worldwide. Projects have been done in England (Sealey), Australia (Golding), Canary Islands (Caparro and Delgodos), France (Picard), Hungary (Gador) and Russia (Menchinskaya) - to mention just a few areas. The following section is a brief account of some of the projects and research results relevant to this study.

Most of the research on manipulative mathematics teaching has been done in elementary schools with Cuisenaire rods (Kieren, 1969). Nelson (1964), Hollis (1965), and Nasea (1966) have supported Gattegno's (1960) theory that Cuisenaire rods increase the student's ability to do computational skills at an earlier age. But Robinson (1964) and Nelson (1964) in a survey of literature on studies with samples containing, in total, 20,000 students from grades one to six in Canada showed that the students

had significantly less ability to transfer knowledge of mathematics to a more generalized setting. To overcome this difficulty of transfer, Dienes believes that a number of different manipulative exercises with different materials should be done for every concept so that the child does not become instrument oriented. Two studies done separately by Williams (1967) and J. Biggs (1967) in England and Wales, have shown that traditional methods of teaching produce good computational results but high anxiety (Biggs, 1967). Cuisenaire and Stern unimodel methods of teaching produced better understanding, motivation, and attitudes than the traditional method; but Dienes multimodel environment produced superior results on all three variables ,(Kieren, 1969).

Further research in activity learning concerned the role of play. Piaget suggested that play allows responses in fantasy which the child cannot make in reality. Anderson (1965) in a study with grade one students found that after twenty-seven sessions with an experimenter, students who played games based on logic were statistically superior to the control group in the number of problems solved, transfer of problems, and the number of trials needed to solve the problems (Vance, 1969).

Several projects have been done in laboratory curriculum development. The best known is the Nuffield Foundation Mathematics Teaching Project, England; directed by G. Matthews. The Nuffield project (1967) found that children like subjects which involve physical activity and an opportunity to talk to other children.

Very little research has been done on mathematics laboratories in secondary schools. Vance (1969) introduced an enrichment program in grade seven and eight, using both laboratory and discovery settings. He

found that the two groups did not differ in regular achievement, although they were doing 25% less normal course work than the control group. These results were consistent with the previous results of Ebeid (1964) and Hopkin (1965) on tests based on concepts taught in labs. Vance found that the test scores favored the discovery class over the math-lab class, alhtough it was not statistically significant. Both groups performed slightly better than the control group, especially on the higher thinking and problem solving questions. An attitude test given to the three groups (control, math-lab., and discovery) showed that there was no significant difference in attitudes towards mathematics. Vance suggested that this may have been because the students regarded the program as being distinct from the regular mathematics course. On the Learning and Doing Mathematics (LDM) semantic differential scale, the math-lab group and discovery class group showed an increase in scores over the control group. This test showed how much the students were enjoying the mathematics, how familiar or real they felt its subject matter was, and how good the tools were for investigating the mathematical principles.

Johnson (1970) did a study similar to Vance's. In a year long study, he studied the attitudes and achievement of six grade seven classes. In his study each teacher taught three types of classes - one method using only a textbook, one method using only activity learning and one using a textbook based lesson enriched with activity lessons. He found that activity learning did not result in improved achievement.

II. Rationale of the Mathematics Laboratory

In the last decade educational research in the field of mathematics has mainly focused attention on the average or above average college-bound student. There is an urgent need for materials for the low achievers. For this reason the investigators* designed a unit of mathematics laboratories on probability in conjunction with the Math 15 program used in Alberta. This unit emphasized the following principles:

1. CONCRETE MATERIALS: The students learned new mathematical concepts by manipulating concrete materials.

2. SOCIAL PARTICIPATION: The students worked in groups of two or three to enable interchange of ideas and thereby speeded up the process of concept attainment.

3. MULTIPLE EMBODIMENT: Employing Dienes' theory of mulitple embodiment, each concept in the unit was taught in several different ways.

4. HEURISTIC QUESTIONS: The lessons were designed to allow the student to "discover" the mathematical concept for himself as much as possible through guided questioning.

5. NOVELTY: Tasks were varied so that the learner was confronted with tasks which were novel and motivating to him.

6. GAME ORIENTED: Dienes' principle of dynamics was incorporated into the lesson. Each lesson had a period of unstructured play followed by a set of questions leading to the "discovery" of a mathematical concept (structured play). For each concept there was a set of questions (practice play).

* Study based on materials designed by Katherine McLeod and John Odynski

7. INDIVIDUALIZATION: Since students work at varying rates, optional lessons were designed for the faster student; that is, not all of the students completed all of the lessons in each section. The lessons were deliberately kept short to help maintain the interest of the student, and wherever possible the student filled in charts. An attempt was made to use everyday language rather than mathematical terms to describe the concepts.

What is a mathematics laboratory? A mathematics laboratory can be defined as a method of mathematical instruction which requires the learner to actively manipulate concrete materials in groups of two or three. Two types of laboratory lessons were developed in the study - the directed laboratory and the non-directed laboratory. The essential difference between these two laboratories was that in the directed laboratory the students were given some mathematical background to the concept usually in the form of a tree diagram before playing the game or solving the problem of the particular lesson. The remaining part of the laboratory lesson was the same for both laboratories.

The following sections will discuss more thoroughly the rationale behind the laboratory lesson. Whenever possible examples of how these principles were incorporated into the lesson are shown.

A. Concrete Materials

Dienes, Bruner and Piaget are advocates of initiating learning by manipulation of concrete materials. As Shulman (1970) says:

> "Piaget's emphasis upon action as a prerequisite to the internalization of cognitive operations has stimulated the focus upon direct manipulation of mathematically relevant materials in the early grades". (1970, P. 65).

In the study, concrete materials such as dice and colored poker chips were made available to the student in each lesson. An attempt was made to keep the equipment as inexpensive as possible and to use the same equipment in as many ways as possible. For example, in one lesson poker-chips represented a blind man's socks and in another lesson downhill ski racers,

B. <u>Social Participation</u>

As discussed in the preceding sections, Piaget believes that active learning can be done through manipulation of concrete materials. He further describes active participation as:

> "When I say 'active' I mean it in two senses. One is acting on material things. But the other means doing things in social collaboration, in a group effort. This leads to a critical frame of mind, where children must communicate with each other. This is an essential factor in intellectual development. Cooperation is indeed co-operation" (quoted by Ripple, 1964, P. 3).

The role of social interaction is important in Piaget's theory of development. When a child participates in a group he becomes aware that another child sees differently from the way he sees. This plays an important role in bringing the child to accommodating (or to rebuilding his point of view) for the new concept. For this reason, one would logically include the students working in groups of two or three in the laboratory setting. Moreover, Skemp has said that the language used by the teacher or the textbook might not be the language available to the student. Interaction with other students might aid the student to understand parts of the lesson which has been written at the wrong language level.

C. Multiple Embodiments

As statedearlier in this chapter, Dienes' principle of multiple embodiments states that is important to use as many different materials and methods to demonstrate one concept, as possible. If the learner becomes too oriented to one piece of equipment, transfer of the concept to equivalent situations is limited. For example, students using Cuisenaire rods in Trinidad were shown to do better in calculations than the conventional method but they were unable to transfer these computational skills to a problem solving situation as they had become too equipment oriented (Vance, 1969).

In the study each concept had several embodiments. For example section 2, permutations, had four embodiments: The Downhill Racer, Jim's Dilemma (photographs), Blind Man and Football Seats (see the sources of reference for the laboratory activities in Appendix A for the complete description of the lessons).

D. Heuristic Questioning

George Polya is the foremost advocate of heuristic teaching (Higgins, 1971). According to him, heuristic teaching is the study of the methods and rules of discovery and invention. He describes heuristics as a list of provocative questions that one should ask himself as he tries to solve a problem. These questions should be constructed in a logical manner in order to elicit discovery. In the study, the lessons were designed to elicit the discovery of a mathematical concept.

An essential feature of heuristic learning is that the principle content of what is to be learned is not given but must be discovered by

the learner before he can incorporate it meaningfully into his cognitive structure. After the discovery, the new cognitive structure is learned through practice in much the same way that the expository method is made meaningful.

The instructions accompanying the mathematic laboratories could have been written using either inductive or deductive reasoning approach. The deductive technique is similar to the expository strategy of teaching which was defined by Eldridge (1965) as rule-example instruction (Vance, 1969). The inductive method of reasoning from specific examples to general rules is very similar to the discovery method of hint-then-formula.

Arguments have been written implying that all that is discovered is meaningful and all that is received (by the expository method) is rote-learned. This of course is not true (Shulman, 1970). Gagne's expository or guided approach of programmed instruction gives specific steps to take the learner through a specified sequence where errors have been minimized and the material has been made meaningful. Bruner, on the other hand, has less system and less order but he still uses structure. The learning begins with the manipulation of materials and tasks that represent the problems. Shulman (1970) summed up the difference between the two men when he said:

> "for Gagne instruction is a smoothly guided tour up a carefully constructed hierarchy of learning tasks; for Bruner, instruction is a roller-coaster ride of successive disequilibrias and equilibria terminating in the attainment or discovery of a desired cognitive state" (1970, P. 53).

Shulman (1970) has made the following chart to describe the continuum between expository lessons and discovery lessons according to

whether the rule and/or solution to the problem is given.

Type of Instruction	Rule	Solution
Exposition	given	given
Guided discovery (deductive)	given	not given
Guided discovery (inductive)	not given	given
"Pure" discovery	not given	not given

(Shulman, 1970, P. 66).

Although Ausubel has agreed that the discovery process is less efficient than the expository approach, he has pointed out that self-discovery has more incentive for the child. Since the sample in this study was low achievers who had previously failed mathematics, motivation was an important reason for using the guided discovery (Inductive) approach in the mathematics laboratory.

E. Novelty

According to DeCecco (1968), four factors affect motivation: arousal, expectancy, incentive and punishment. The first, arousal, is the general state of excitability of an individual. The optimal level of arousal for the most efficient functioning is the intermediate level between boredom and excitement. Boredom arises from a monotonous environment or the performance of repetitive tasks. In Berlyne's (1957) study of human behavior, he found that subjects indicated a higher performance for incongruous rather than normal pictures. For this reason, the laboratory lessons introduced the new mathematical concepts in a novel problem setting. For example, in the lesson entitled "Blind Man"

the following problem was posed:

> A blind man wishes to go on a trip. There is no one there to help him pack his suitcase. When he reaches into the drawer, he realizes that his socks have not been paired. He had six socks, two white, two black and two red socks. How many socks must he take with him in order that he has at least one pair of socks?

Fowler (1965) views curiousity as an acquired expectation. He feels that people will not be curious about stimulation of which they are unaware (DeCecco, 1968). To ensure familiarity with the blind man problem, a bag, two white poker chips, two black chips and two red chips were available to the student to manipulate and test to see if his hypothesis was correct.

The momentary belief that a particular outcome will follow a particular act describes the expectancy of an individual. In the example, the child's hypothesis would be the expectancy of the individual and his incentive to produce the right answer can be explained by the excitation of the novel problem or the competitive feeling of producing the answer before his laboratory partner does. Incentives are the concrete or symbolic rewards one receives by achieving actual goals whereas punishment is a stimulus which a person may wish to avoid.

F. Game Oriented

Dienes' Principle of dynamics can easily be applied to a mathematical laboratory. Basically he divided the process of concept attainment into three types of games. The first, the "preliminary game", consists of largely unstructured play with concrete materials. This is the type of manipulative play found in mathematics laboratories in the study. Next

is the "structured game" or rule-bound play. In the study the structured game corresponds to the questions which follow the unstructured play. These questions lead to the abstraction of the mathematical concept behind the game. The final type of game is a "practice game" which serves to anchor the insights of the structured game. In the study, a question sheet after each concept acts as the "practice game".

On the following page is the sample laboratory lesson - the "Grasshopper Game"*. The first portion of the laboratory lesson involves playing the "grasshopper game" (the preliminary game). Next there is a set of questions leading to the insight of the mathematical concept behind the game (the structured game). Following this laboratory, there are several questions which fullfill the purpose of the "practice game".

* Adapted from "Activities in Mathematics", published by Scott Foresmont & Co. Inc., 1971.

GRASSHOPPER GAME

Materials: 3 different coins: penny, nickel, dime

The circles and squares are represented by letters at the bottom of these positions.

Two people are to play this game.

Rules:

1. Flip a coin, the winner chooses the position he prefers (either circle or square).

2. Each play begins at "O" (START) and then toss the three coins (penny, nickel, dime).

3. Notice the way the coins land, for each head move one to the right, for each tail move one to the left (i.e. if coins land THH, your moves would be O to W to O to A and therefore one point for squares. If on next toss coins landed HHH, your moves would be O to A to B to C and a point for the circles).

4. One person is to flip the coins, the other is to record the results in Table 1.

5. 16 plays (16 tosses of 3 coins) make up a game.

6. Play 3 games.

7. After each game, total your scores. The one with the most tallies wins.

8. NOTE: Frequency means the number of times something occurs.

TABLE 1

		GAME 1			GAME 2			GAME 3		
		Tally	Fre-quency	Score	Tally	Fre-quency	Score	Tally	Fre-quency	Score
Squares	A									
	W									
Circles	Z									
	Y									
	X									
	B									
	C									
	D									

TABLE 2 (For all 3 Games)

Place Play Ends	Frequency (Total for 3 games)	Fraction of Total Plays
A		
W		
Z		
Y		
X		
B		
C		
D		

Total =

The fractions should add up to 1.

1. Do you think this game is fair? Why?

2. If you played this game again, what shape would you choose? Why?

3. Why is the frequency for each Z, X, D, B = 0? Explain.

4. What are the different ways in which the three coins may land? List them.

5. What are 6 ways out of a total of 8 possible ways the coins land that will lead to squares getting a point?

6. Probability is defined as $\frac{\text{no. of favorable outcomes}}{\text{total no. of possible outcomes}}$.
Calculate the probabilities for obtaining each of Z, Y, X, W, A, B, C, D and compare these with the ratios in Table 2. Have you any explanation for the differences? Since you are calculating the probabilities for all possible outcomes, the probabilities should add to 1.

7. For 128 trials in the Grasshopper game, and using your results from probabilities in question 6, how many trials would end at:

(a) W ____________	(e) A __________
(b) X ____________	(f) B __________
(c) Y ____________	(g) C __________
(d) Z ____________	(h) D __________

Did you get 48, 0, 16, 0, 48, 0, 16, 0? Do these add up to 128? What conclusions can you make?

8. How does the total of your scores (from Table 2) in the ratio number of squares to number of circles compare to the calculated probabilities of $\frac{A + W}{Y + C}$? If these ratios differ, give an explanation.

9. How would you make this game fair?

G. Individualization

Johnson and Rising (1967) listed several individual differences which are of significance in learning mathematics. These include:

1. THE MENTAL ABILITY OF THE STUDENT AND THEIR ABILITY TO THINK REFLECTIVELY. In the sample the I.Q. score ranged from 86 - 134 even though these students had all failed their previous years work in mathematics. Equally diverse are their Skemp scores which measures their

reflective intelligence. Out of a possible score of 40, the range was from 0 to 31. Ideally, these measures of ability should indicate the differences in the learning rate as well as in the quality of their responses.

2. MATHEMATICAL ABILITY SUCH AS THE ABILITY TO USE SYMBOLS, THE ABILITY TO DO LOGICAL REASONING OR THE ABILITY TO COMPUTE: The mathematics laboratories were designed to avoid the use of symbolism. It was felt that some of the difficulty which the students were having in learning mathematics might be due to their particular stage which they were at. Chronologically they should have been well into the formal operations stage but if earlier concepts were rote-learned (without understanding) the basic cognitive structure to build new concepts would not be available. Since the ability to compute is very important in everyday activities an effort was made to have the students do as much computing as possible.

3. KNOWLEDGE OF MATHEMATICAL CONCEPTS, STRUCTURES AND PROCESSES:

The previous training of the students in the sample had not been successful. For this reason concrete materials were made available so that the student might be able to construct concepts which were missing in their cognitive structure.

4. MOTIVATIONS, INTERESTS, ATTITUDES AND APPRECIATION:

This is the topic which is discussed in a thesis associated with this study (see the work of John Odynski).

5. PHYSICAL, EMOTIONAL AND SOCIAL MATURITY OF THE LEARNER:

Student variations are diverse so that a program of individualized instruction which takes into account some of these variables is advisable.

6. SPECIAL TALENT OR DEFICIENCIES SUCH AS CREATIVITY, LACK OF READING OR RETENTION SPAN: Since the students in the sample were low achievers, reading skills are often a problem. For this reason everyday language was used instead of mathematical terms and an attempt was made to keep the lessons short since the low achiever often has a short attention span. The low achiever can also have a low retention span, therefore, many embodiments of each concept were given with the hope that after several times of being in contact with the material, the student might retain the concepts.

7. LEARNING HABITS SUCH AS SELF-DISCIPLINE AND ORGANIZATION OF WRITTEN WORK: The learning habits of the sample were obviously poor, therefore an attempt was made to keep their work organized by having them fill in charts.

H. Advance Organizers

Both Ausubel and Gagne advocate a carefully guided expository sequence in learning new concepts but they offer different arguments on the order of sequencing material (Shulman 1970). Gagne recommended that the instructional sequence proceed from the less abstract to the more abstract material while Ausubel advocated what he describes as "progressive differentiation". According to Ausubel, progressive differentiation is the sequencing of material from a higher level on the hierarchy of abstraction to a less abstract level, the level at which the material is to be mastered. He coined the term "advance organizer" to describe the initial material at the abstract level.

An advance organizer is a set of organizing statements which initiates a unit of instruction. It is a higher level of abstraction than what must be subsequently learned; therefore, it is used as an "ideational scaffolding" for the later work. Ausubel (1963) suggested that advance organizers help to maximally stabilize and discriminate between the related conceptual system in the learner's structure. The advance organizer economizes learning since it points out the way previously related concepts are either basically similar or essentially different from the new concepts being taught.

In the study, the directed laboratory differed from the non-directed laboratory in that the directed laboratory had additional development placed at the beginning of the lesson. This development was similar to an advance organizer since the student was able to use the introduction as an "ideational scaffolding" on which to build the new concept. Advance organizers were used in the study since, Ausubel (1963) found that they enable the student to optimize retention and transfer of learning.

According to Dienes' principle of dynamics, the directed laboratory has less emphasis on the period of 'unstructured' play than the non-directed laboratory since the advance organizer gave the framework for the lesson. In other words, the directed laboratory has more structured play while the non-directed laboratory has a longer period of unstructured play followed by a period of structured play.

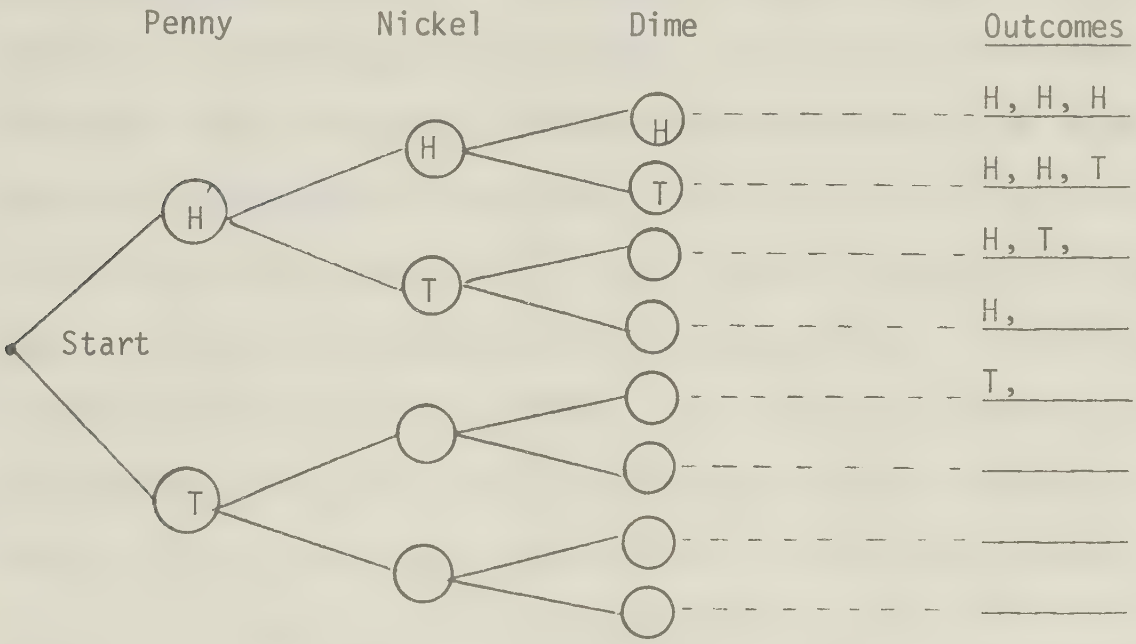
Ausubel has distinguished two types of advance organizers. The first, the 'expository' organizer is a set of organizing statements which are given to the student who is completely unfamiliar with the material. The second, the 'comparative' organizer which is given to the student who

is familiar with the concept. The comparative organizer is used to integrate the new concept with basically similar concepts from the child's cognitive structure as well as discriminate between the new and existing ideas which are essentially different and easy to confuse. Since the students in the study were familiar with concepts related to the new concept, the advance organizer used in the directed mathematics laboratory is similar to the "comparative" organizer. In the laboratory lessons, most of the "advance" organizers were in the form of tree diagrams. A tree diagram is a diagrammatic way of representing a complicated relationship between objects. For example the grasshopper game used a tree diagram of the possible ways of flipping three coins. On the next page is the introduction used in the directed laboratory "Grasshopper Game".

GRASSHOPPER GAME

Materials: 3 different coins; penny, nickel, dime

1) What are the possible outcomes when 3 coins are tossed at a time (penny, nickel, dime)? Below is a tree diagram which is partially completed. Fill in the rest. If you are having problems, toss the three coins and notice possible outcomes.



2) The fraction of heads on penny, tails on nickel, tails on dime or (H, T, T) to the total number of possible outcomes is 1/8. Ratio of number of (H,H,H) to total number of outcomes possible is: ________. Ratio of number of (H,H,T) to number of possible outcomes __________. Is the ratio of number of (H,H,T) to the number of possible outcomes the SAME as the ratio of the number of 2 heads and a tail (in any order) to the total number of possible outcomes? Why not?

There are several studies which have shown evidence on the effectiveness of advance organizers. Ausubel (1960) did an experiment using an expository organizer in a study about the properties of steel. The experimental group was given an advance organizer which emphasized the similarities and differences between metals and alloys, and the advantages and disadvantages of alloys. The control group was given a historical introduction on metallogy such as those found in the introduction of a textbook. The results were statistically significant. The experimental group had a mean score of 16.7 while the control groups score was 14.1. Later, Ausubel and Youssef (1963) did a similar experiment with two passages in sequence about Buddhism and Zen Buddhism. The advance organizer pointed out the differences and similarities between Christianity and Buddhism and a similar distinction between Buddhism and Zen Buddhism. The control group was given historical and biographical material prior to the two passages. The results were shown as below:

Group	Mean Buddhism Tests Scores	Mean Zen Buddhism Test Scores
Experimental	19.4	14.8
Control	17.6	14.2
Significance of difference	significant	not significant

The non-statistical significance between Zen Buddhism test scores can be explained by the fact that the Buddhism study acted as an advance organizer for both groups.

Ausubel and Fitzgerald (1962) found that expository organizers aided significantly the learner who had low verbal and low analytic ability (Schultz (1966)) and hence a learner who had presumably less ability to

develop adequate schema from their own existing cognitive structures. Since the sample in the study involved the low achievers and since one of the characteristics of a low achiever has been described as low verbal ability (Johnson and Rising, 1967), it would appear from the research that advance organizers might aid the low achiever in learning a new concept. Other research by Ausubel and Fitzgerald (1965), Merrill and Stolurow (19661), Newton and Hickey (1965), Grotelueschen and Sjogren (1968), and Scandura and Wells (1967) obtained results substantiating Ausubel's theory (Ausubel 1963).

In conclusion, the direct laboratory differed from the non-directed laboratory since the directed laboratory used a "comparative" organizer to introduce each new lesson. This meant than the direct laboratory lesson had less emphasis on the "unstructured" game period as described in Dienes principle of dynamics.

III. Outline of the Laboratory Material

The following is a brief outline of the activities used in the laboratory lessons in the study. There were four different sections incorporated into the unit on probability from the Math 15 program. Each section emphasized one or two of the six concepts developed in the unit. These concepts were counting outcomes, permutations, combinations, probability, independent events, and dependent events. Preceding the first section was a sample laboratory which was used as a training lesson. It enabled the students and teachers to become familiar with the mathematics laboratory technique before actually starting the experimental program. See Appendix A for the source of the materials adapted in the lessons.

INTRODUCTION - Probability

This sample lesson introduced the concept of probability. In essence the laboratory lesson through the manipulation of an ordinary die and a colored block tried to lead the student to question whether a game is fair or unfair - a fair game being one in which all players have equal chances of winning.

SECTION I - Counting Outcomes

There are four lessons in this section and each lesson teaches the concept of counting the possible outcomes of an event. This section is basic to the remaining sections since the idea of representing the outcomes in tree-diagrams was developed.

A. Pick Your Choice

The materials used in this game consist of a gameboard made of pink and blue squares. The players in the game were told to toss two dice and to make the same number of moves on the gameboard as the sum of the dots on the two dice. If a player landed on a pink square one point was given to the pink team and if the player landed on a blue square, one point was given to the blue team. Through free play and later by directed questions the students learned how to count the outcomes of an event. (See Appendix B for complete lesson.)

B. In Jail

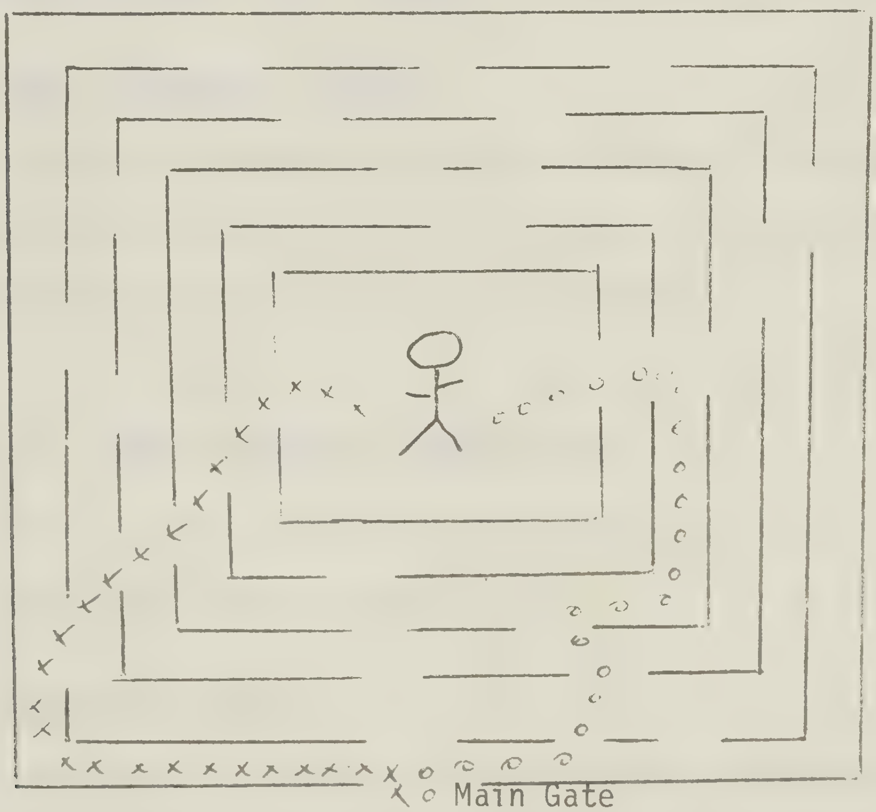
The second laboratory lesson on counting involved the solving of a problem. The problem was as follows:

When the judge was sentencing Charlie Brown for speeding, he gave him two choices:

a) the city jail for one year or;

b) the jail at Pineville.

Pineville jail is rather different. Charlie is given a chance once a day to try to escape. There are many paths out of the prison at Pineville jail. Two of the possible paths are shown below the diagram. Charlie can escape through only one of these paths. If he chooses or follows the wrong path, the main gate closes and he must return to the center of the prison and try to escape again the next day. Which jail should Charlie Brown chose and why?



Since the question is rather difficult to answer without some method for counting outcomes, the students were lead through directive questioning to developing a teachnique for solving this problem. Following this lesson is a practice session pertaining to counting outcomes.

C. Chain Letter

This lesson involved counting the possible outcomes of a chain letter. Briefly, the problem was presented in the following way: How can you make $8,000 by spending only one dollar? The students were given a sample chain letter which described how they theoretically could make $8,000 by spending one dollar. By manipulating colored chips they were able to find the answer to simplified questions and later they generalized these ideas to answer the initial question. Following this lesson was a problem set pertaining to counting outcomes.

D. Pascal Triangle (Optional)

The Pascal triangle lesson was optional. The students were given four different coins and told to flip them. The principle behind the game was developed around the Pascal Triangle.

SECTION II - Combinations and Permutations

There are five lessons in this section and each lesson tries to teach permutations and/or combinations.

A. The Downhill Racer

This lesson is based on the advantages of being the first skier to compete in a downhill race. The student's problem is to find out how many different ways four skiers can race down the hill. By manipulating poker chips the student actually demonstrates how permutations work.(see Appendix B)

B. Jim's Dilemma

This lesson is based on the following problem:

Five boys are being photographed. They can not seem to decide in

which order they should stand to be photographed. After several minutes of arguing one boy said, "Gosh, if we keep rearranging ourselves, we might be here for two hours". If each time they rearrange themselves, it takes a minute, would the boy's statement be correct?

Through manipulating chips and colored cards, the student is able to uncover the mathematical idea developed in the lesson. Following this lesson was a set of practice questions on permutations.

C. Blind Man

By manipulating poker chips, the student is able to find the answer to the following problem:

A blind man wishes to go on a trip. There is no one there to help him pack his suitcase. When he reaches into the drawer, he realizes that his socks have not been paired. He has six socks, two white, two black and two red socks. How many socks must he take with him in order that he has at least one pair of socks?

D. Football Seats

Again this lesson is based on a problem. Briefly, four girls went to a football game and they had trouble deciding in which order to sit. This lesson developed a mathematical concept of combinations. Following the lesson was a problem set pertaining to combination questions.

E. Station 13 - OPTIONAL

This lesson is based on a problem setting. A gasoline company was giving free gifts to anyone who could open a lock on a treasure chest. The problem was to find out why at one particular station (Station 13) the owner was losing more money from the treasure

chest gimmick than he was gaining from the publicity. The trouble had to do with the combination lock which was not functioning correctly. Through manipulating colored chips and cards, the students were able to demonstrate what was wrong with the lock. Following this lesson was a set of problems pertaining to permutations and combinations.

F. Circular Arrangements - OPTIONAL

This lesson introduced circular permutations which are different from the regular permutation. This lesson was developed around the question of how many different ways four people can be arranged around a table. Following this lesson was a problem set pertaining to combinations and permutations.

SECTION III - Probability

There are three lessons in this section and each lesson introduces the basic principles of probability.

A. Grasshopper Game

This lesson teaches the principle of probability through a game setting. Briefly, two people play this game. The students flip a coin to decide whether or not they choose the circles or squares from the gameboard. Since there are six circles and two squares on the gameboard, a student without thinking about the principles behind the game would automatically choose the six circles as having more chances of winning. After playing the game the student learns that he must analyze the principles behind the game before playing it to see if the odds are in favour of winning (see page 40 for the complete lesson).

B. Football Game

This lesson is similar to the Grasshopper Game. Again the game teaches probability on the basis of the way three different coins can land. When a coin lands heads,the player advances down the field and when the coin lands tails, the player loses territory. Depending on how far he advances the player will get 0 to 7 points. Following this lesson are several questions pertaining to probability. (See Appendix B)

C. Bag It

This game was based on the ability of a person to predict the number of white chips in a bag of 20 black and white chips. After taking a sample of 4, 8, 12 or 16 chips, the student was asked to guess what percentage of the chips were white. Scores were given every time a prediction was correctly made.

SECTION IV - Dependent Events and Independent Events

There are four lessons in the section. The first two lessons introduce the principle of dependent events and the remaining lessons incorporate both independent and dependent events.

A. Die vs. Coin

In this game the players are given a choice of winning by tossing a die twice and getting four or less on both tosses or by flipping a coin twice and getting a head on both flips. The scores are recorded in a table and from this table the student can observe which method has more chances of winning.

B. What Color?

This game teaches dependent events. The object of the game is to pick two chips of the same color from a bag of five white chips and five blue chips. After picking out the first chip, it is not returned to the bag until the second chip has been drawn.

C. Red, Orange or Blue, Green, Yellow

In this game ten red cards, two green cards, six blue cards, two yellow cards and five orange cards are placed into a bag. A coin is flipped and the winning player chooses the route that he prefers to follow in scoring a point. The two routes were picking a red, or orange card or picking a blue, green or yellow card. This game demonstrates the difference between dependent and independent events. Following this lesson is a set of problems on independent and dependent events. (See Appenix B)

D. 6 or under, 7 or over

There are two different routes in this game. After flipping a coin, the winning player has a choice of scoring a point when the total number of dots on two dice is six or less, or seven or more. Again, this lesson demonstrates the difference between dependent and independent events.

Review Exercise

Following the last lesson on independent and dependent events, is a review sheet of thirty multiple choice questions on the six concepts taught in the unit.

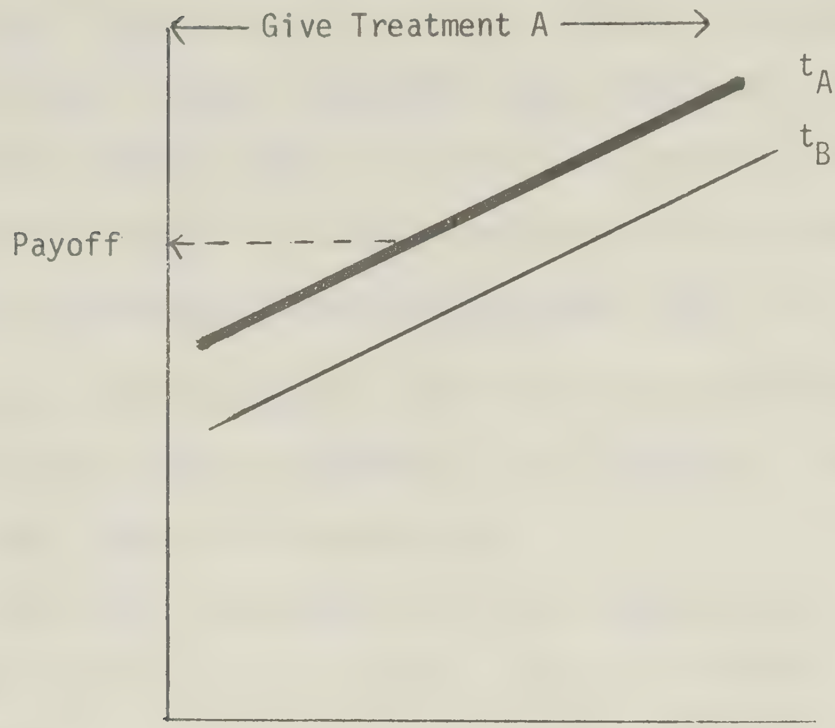
CHAPTER III

A REVIEW OF LITERATURE RELATED TO THE RESEARCH PROBLEM

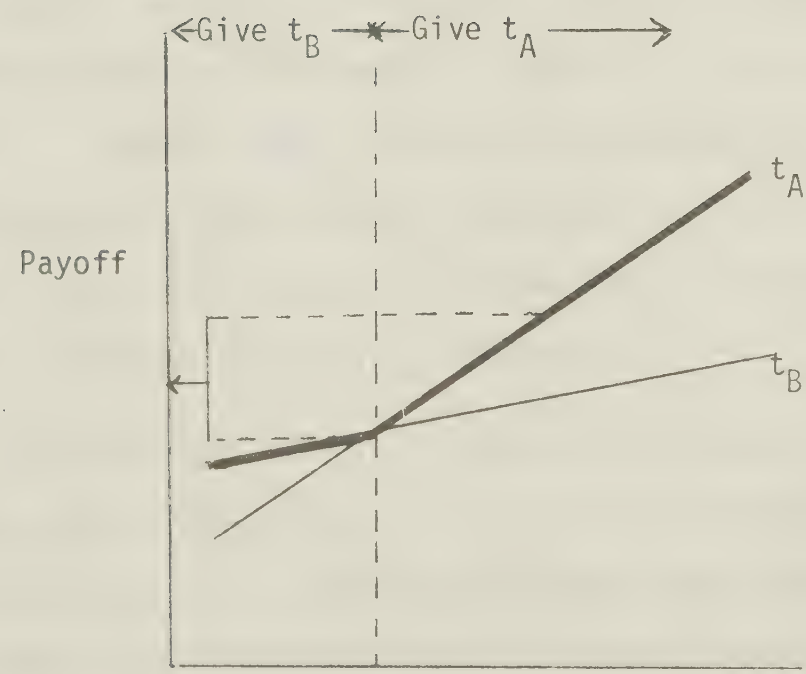
The directed and non-directed mathematics laboratories discussed in Chapter II were designed as individualized programs of instruction. One can recall from Chapter I that Mitzel (1970) differentiated between individual instruction and individualized instruction. The former consisted of a program tailored for the individual in isolation while the latter suggested that some variable or variables (such as an aptitude variable) had been taken into account in building the program. Becker (1970) made a strong case for studying aptitude-treatment interactions. Since there any many individual modes of learning, some individuals achieve better using method X while others achieve better using method Y, and since most studies have been interested in only main effects (such as is method X a better method of teaching than method Y), Becker advocated a study on aptitude-treatment interaction as a method of developing a theory on how mathematics can be taught most efficiently.

Cronbach (1957) gave a further explanation to why aptitude-treatment-interaction research is necessary for more truly individualized mathematic programs. He found that personality variables interact with methods of instruction. For example two treatments (t_a and t_b) can have many payoff functions. Figure I shows the payoff for two treatments using the mean difference between treatments and a valid aptitude predictor. Although the aptitude variable is a valid predictor, it is useless since treatment A is superior in all cases. Figure 2, on the

Payoff Functions for Two Treatments



Aptitude - Figure 1



Aptitude - Figure 2

other hand, suggests that some individuals take treatment A and some treatment B. This particular aptitude variable is a better discrimination. In Figure 3, three treatments were introduced. Treatment C would have been the best treatment to assign to the sample if only one treatment was available but if all three programs were available, the treatment assigned would depend on the individual's aptitude score.

Becker (1970) suggested that there were several areas of difficulty that could be encountered in an aptitude - treatment - interaction model. These difficulties were:

1. The selection of aptitude measures to measure specific aspects of mental abilities (in the study, Skemp's and Kagan's variables were tested).
2. The length of the treatment period (in the study there were 1,000 to 1,600 minutes of laboratory lessons).
3. The impact of past instruction. Snow, Tiffin and Siebert (1965) and Brownel and Moser (1949) showed that the subject will respond to the new treatment as a function of past experience (Becker 1970). (In the study the students in the sample had previously failed Grade 9 mathematics).
4. Type of achievement measure. (In the study a thirty question multiple choice achievement test was administered).

The following sections of Chapter III discuss the two aptitude variables used in the study. A review of the literature, a description of the tests used in the study and the theoretical framework for the design hypothesis is developed for both Kagan's variables of reflective and impulsive personalities and Skemp's variable of reflective intelligence.

Payoff Functions for Three Treatments

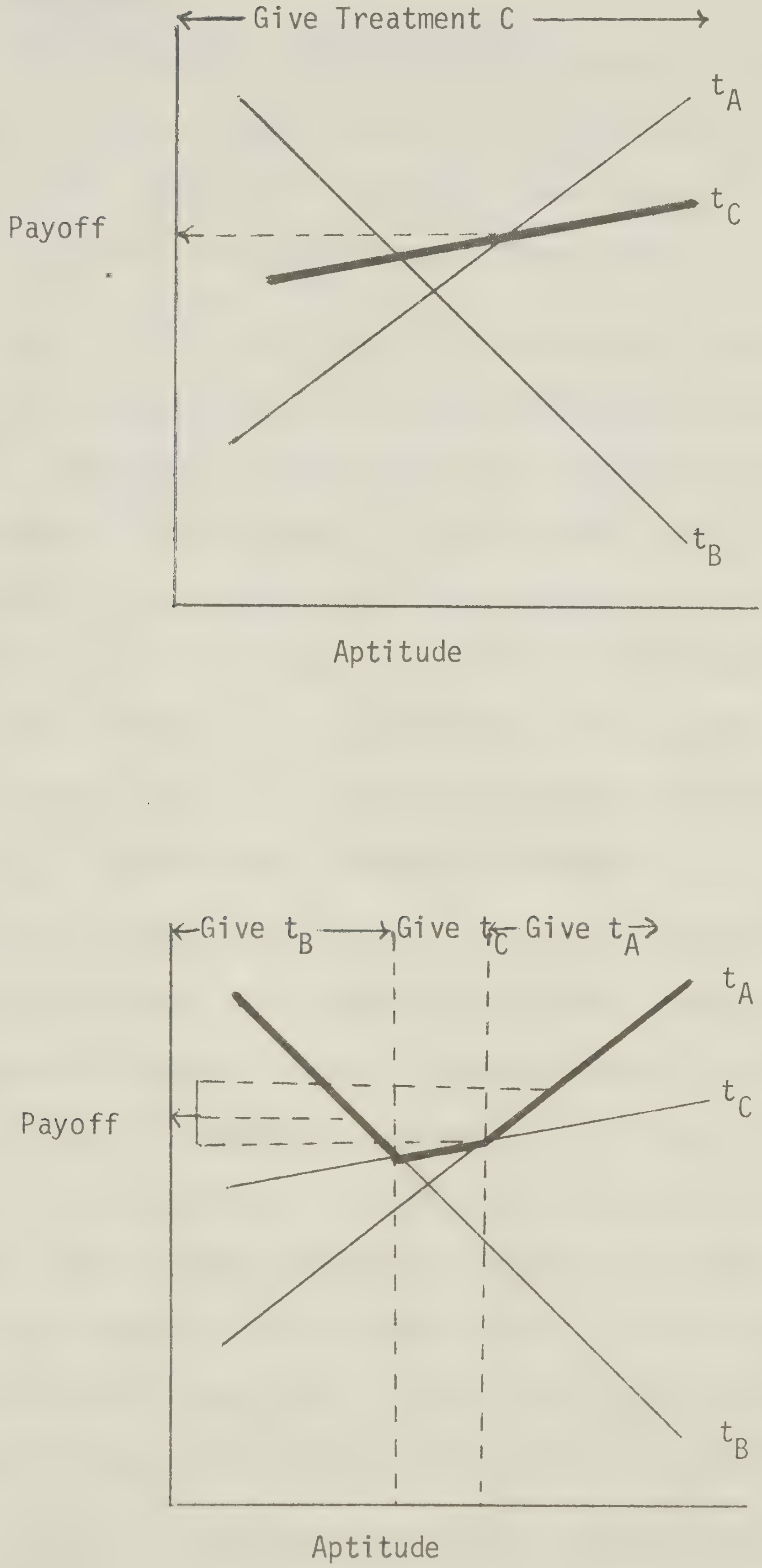


Figure 3

I. Impulsive and Reflective Thinking

A. Background

The differences in problem solving amongst children has been attributed to motivational variables and conceptual variables (Shulman 1970). In this study, the investigator was interested in Kagan's conceptual variable of impulsivity and reflectivity*.

Kagan's (1963) contribution to conceptual development was a demonstration that there are individual differences in the time it takes to solve a problem. He coined the term "conceptual tempo" to describe the tendency of the individual to spend either long or short decision times when solving problems which have several possible solutions. The Matching Familiar Figures Test (MFF) was developed by Kagan to measure this response in time difference. The test was constructed in such a way that there are a number of solutions to a problem simultaneously available. This guaranteed "response uncertainty".

Individuals who make rapid, non-analytic responses are described as impulsive thinkers and individuals who make delayed analytic responses as reflective thinkers. Kagan's research uncovered that the amount of response time is inversely proportional to the number of errors made in response to each problem. In other words, impulsive persons were found to make a larger number of errors in inductive reasoning problems because they did not pause to evaluate the quality of their inferences whereas reflective persons were more critical of their solutions to the problems and therefore made fewer errors. This raises a question of whether there

* See the thesis written by John Odynski for the motivational aspects of this study.

is any difference between the thoughts or number of thoughts of an impulsive or reflective student or if the child just sitting immobile in his seat. Kagan (1966) measured the number of times the child oriented his head and eyes toward the problem to try to answer this question. Results of analysis of eye fixation on the standard indicated that the reflective individual was not just sitting paralyzed in his seat. Only a few showed strong inhibitions in offering any response perhaps out of fear of responding to a strange adult. Kagan also found that about every three seconds the child re-oriented his head. This tendency was found to be invariant over response latency, age and sex of the child. Kagan (1966) suggested that the need for action may cause the impulsive child to blurt out answers before reflecting about them. The impulsive child should not be dismissed as having insufficient knowledge or lacking in understanding. Perhaps the impulsive child should be taught to be reflective by teaching in a setting in which there is no premium on time, or it may be that an impulsive child will fare better in a learning setting which presents him more limited alternatives. Since teachers often associate speed with intelligence and since the sample in this study was low achievers, some students might have become geared to making rapid responses. On the other hand, constant failure might make other students very anxious about making a mistake and therefore make them appear to be reflective students.

B. Instrument

Kagan developed the Matching Familiar Figures Test (MFF) to test the latency - error relationship of reflective and impulsive thinkers. In the adult version of the MFF, the tester shows the subject a picture

of a familiar object and asks the subject to choose an identical picture from eight variants (see the Appendix E for test sample). The children's version uses the same standards but has only six variants. The time which it takes the subject to decide his first response is recorded to the nearest half second along with the number of errors that are made in answering the twelve questions correctly.

The MFF appears to be the most sensitive instrument for measuring impulsivity and reflectivity (Kagan 1965). To select the purest groups of impulsive and reflective subjects in a large sample, Kagan (1966) suggested that subjects scoring above the median on response time and below the median on number of errors be classified as reflective and those scoring below the median on response time and above the median on number of errors be classified as impulsive.

The MFF test is the most commonly used to test for response latency because it has been shown to be reliable. For example Kagan (1965) in a test-retest situation showed that the stability of response time over a period of one year with a sample of 102 grade one students. The response time correlated in the sample of 46 boys and 56 girls were .48 and .50 respectively ($p. < .01$). The error scores over a period of a year correlated poorly for the boys but satisfactory for the girls ($r = .25$ and .51 respectively). Yondo and Kagan (1968) in the adult version of the MFF test found that it discriminated well amongst a group of primary teachers. They found that there was virtually no overlap between reflective teacher's response time and the impulsive teacher's time - the fastest reflective teacher's response time being twice that of the slowest impulsive teacher. Ward (1968) extended the study to kinder-

garten. His work offered further support for reflectivity and impulsivity as a dimension of individual differences in cognitive style.

C. Research Findings

It is of educational significance to determine whether impulsive children are at a disadvantage because of their numerous errors. Kagan (1966) found that impulsive subjects made more reading errors than reflective subjects and that reflective subjects were better at solving inductive reasoning problems. Campbell (1968) found that there was no significant difference in the performance of impulsive and reflective grade six boys on the Iowa Scholastic Schievement Battery. Cathcart and Liedtke (1969) on the other hand, found that with grade two and three students the reflective student scored significantly higher on mathematics achievement tests than did impulsive students. Gupta (1970) also showed that the reflective group had a significantly greater persistence and showed better achievement in school subjects. Souch (1970) with a sample of five hundred students from grades one to six found that impulsive children performed significantly more poorly than reflective children on mathematics achievement tests but no significant differences were noted for standardized reading tests. In the same study, Souch found that there was no tendency for children to become more reflective individuals with respect to latency response as they became older. Unlike Souch's findings, Kagan's (1966) earlier report suggested that reflection and accuracy tends to increase with age. It is unlikely that the MFF test effected the results since the tasks should become easier for the older children.

D. Theoretical Framework

"Response uncertainty" is considered present when difficult items force a child to mentally generate his own alternative solutions or hypotheses. Kagan (1968) found that impulsive children make more errors in tests involving tasks containing response uncertainty. It would seem reasonable to hypothesize that reflective students should achieve better than impulsive students in both the non-directed and directed laboratories when taking into account the findings of Kagan (1966), Cathcart and Liedtke (1969), Gupta (1970) and Souch (1970. The directed laboratory will probably be the better of the two settings for the impulsive student since this type of lab had advance organizers to help orient the impulsive student and therefore there should be less response uncertainty. The non-directed lab, on the other hand, should be better for the reflective student since the questions require more thought.

II. Reflective Intelligence

A. Background

Skemp (1958) formulated a three-part comprehensive theory of mathematics learning. Since mathematics learning is hierarchically dependent, Skemp's theory dealt with highly organized systems of thinking. His theory attempted to answer the question: What does a child need besides general intelligence to succeed in mathematics? Skemp hypothesized that if an intelligent child performed poorly in mathematics it was not necessarily because he had not formed the concepts and operations necessary for success but rather that the child could not reflect on these

concepts. His three-part theory states:

1) that mathematical concepts can be learned efficiently.

2) that there is a schematic method of learning mathematics and

3) that mathematics learning depends on reflective use of intelligence.

To understand what is meant by reflective intelligence one must understand Skemp's theory of concept attainment. The primary level of concepts can be exemplified in physical objects and actions while the secondary level consists of concepts which can only be symbolized. At the primary level there are two types of intelligence - sensory intelligence and motor intelligence hence Skemp refers to concepts at the level as sensori-motor concepts. He describes sensory intelligence as an "awareness of certain relationships between sensory stimuli ... as distinct from (and often opposed to) resemblance between sensory stimulus" (Skemp, 1961, P. 47). For example a child has learned the sensory concept "four" when he as understood the relationship between four lamps, four tables and four chairs. Motor intelligence on the other hand, "involves an awareness of relationships between actions such as filling up and emptying out, putting together and taking apart, taking away and putting back" (Skemp, 1961, P. 47). Therefore, sensory-motor intelligence is understanding relationships between objects, groups of objects and one's action with these objects. Arithmetic is an example of a concept using sensory-motor intelligence.

Algebra, on the other hand, involves secondary levels of concept attainment. To form secondary concepts, one must be able to think reflectively. Piaget (1950) described three conditions for the transition from the sensory-motor concept level to the reflective concept level. The

first, is the increase in speed which moulds successive phases of action into a simultaneous whole. The second is the conscious search for the solution of a problem and for the understanding of the nature of the problem. And, finally, the third is the action of taking the physical concept to symbolism and to act on symbolism. Skemp (1961) defines reflective intelligence as a second order system which:

> (i) can perceive and act on the concepts and operations of the sensory-motor system.
> (ii) can perceive relationships between these concepts and operations and,
> (iii) can act on them in ways which take account of these relationships and of other information from memory and from the external environment (1961, P. 49).

In other words, reflective intellgience is the ability of the mind to become aware of concepts and the ability to manipulate its own concepts. An example of reflective thought is the type that is required to solve a problem which can not be solved by routine application of a concept but which requires some combination or modification of existing concepts.

B. Instrument

Skemp (1958) developed two tests which were designed to measure reflective intelligence. The first test, SK4, was developed to measure the subjects ability to form concepts (Part I) and to manipulate these concepts (Part II). In the first part of SK4, fifteen questions involving properties of a simple line (such as being straight, curved, dotted or continuous) were designed to test the subjects ability to form a particular class-concept. Each question consisted of three "example" figures which belong to the same set, three figures which were "not example" of the previous set and three "test" figures. The subject was asked to tell which "test" figures were members of the "example" set. Since the second

part of the test involved the manipulation of the concepts developed in the first part, the first part was designed to maximize scores. Skemp did a pilot study with twelve year olds to eliminate difficult questions from Part I.

The second part of SK4 was designed to measure the students ability to manipulate concepts at a reflective level. In this part, thirty-five pairs of properties used in the first part were chosen and three "examples" of each of these pairs of properties were drawn. Three "not example" figures were also drawn with one or both of the properties missing. The subjects were asked to decide whether or not each of the three "test" figures possessed both of the properties of the "example" figures. To be able to respond correctly to the question the subject must think reflectively (see Appendix E for test SK4).

The second test, SK5, was designed to measure the students ability to perform operations (Part I) and ability to manipulate these operations (Part II). SK5, Part I consisted of a question sheet and a demonstration sheet giving three simple abstract-line-figure examples of fifteen operations (such as clockwise rotation through a right angle, or a reflection in a horizontal line). The subject was asked to examine the operations on the demonstration sheet and then to carry out the operation shown in the example on three specified figures on the question sheet. Again Part I was designed to maximize scores. The second part of the operations test was designed to measure the ability to manipulate operations involving combining and reversing the operations of Part I. The operations in the second part were appropriate to a mathematics problem since practically every mathematics operation has an inverse (reverse) and success in problem

solving depends on ones choice of appropriate operations and combining of these operations. The first five questions involved the combination of two operations (one after the other) from the demonstration sheet and drawing the resulting figures. The second five problems involved carrying out the reverse of a single operation and the third group of five involves first the combining and then reversing of the operations.

Before beginning Part II of SK5 the subjects were given the results to Part I (after their answers had been collected) and these were explained to them to ensure that the subjects understood the operations. The demonstration sheets were used in both parts of the test so that no memory work was necessary. (See Appendix 3 for a modified version of SK5. It is labelled SK6 Part I and Part II.)

The results from a study by Skemp (1958) of a sample of fifty students of the fifth form who wrote SK4, SK5 and also the general certificate of education (G.C.E.) mathematics exam are shown in Table I on the next page. The highest correlation was between the manipulation of Operation SK5 Part B scores and the mathematics scores (.72). This supports Skemp's theory that reflective intelligence is an important variable in mathematics achievement and therefore in this study, SK6, the modified version of SK5, was administered to the sample.

C. Research Findings

Harrison (1969) did his doctorial dissertation on Skemp's reflective intelligence. He questioned whether reflective intelligence was actually a factor important for success in mathematics. Harrison found that in a sample of 340 students from Grades five to eleven, that

CORRELATIONS AMONG SCORES OBTAINED BY FIFTY FIFTH FORM STUDENTS ON SKEMP'S TESTS AND ON A.G.C.E. MATHEMATICS EXAM*

	SK4, Part 2	SK5 A	SK5 B	G.C.E. Mathematics
SK4, Part 2	1.00	0.66	0.68	0.58
SK5 A	0.66	1.00	0.58	0.42
SK5 B	0.68	0.58	1.00	0.72
G.C.E. Mathematics	0.58	0.42	0.72	1.00

* R.R. Skemp, "An Investigation into Difficulties in Learning Mathematics by Children of Good General Intelligence" (unpublished Doctoral Dissertation, The University of Manchester, 1958 , P. 162)

reflective intelligence as measured by Skemp's tests made a significant contribution to the prediction of mathematics when added to a prediction equation derived from the general intelligence scores. He also found that there was no significant difference in performance between boys and girls and that test anxiety was not an important factor in the success of mathematics. He was able to show that the 15-16 year old student achieved significantly higher scores than 10-12 year old students. This supported Piagets and Skemps description of adolescent thinking as being qualitatively different from that of younger children (Harrison, 1969).

D. Theoretical Framework

Skemp's variable of reflective intelligence is a separate variable from that found in general intelligence tests (Harrison, 1967). It is used to describe the characteristics of a person who is mathematically inclined. The investigator tested to see if there were any significant differences between Skemp scores and achievement scores in the two laboratories when I.Q. is an invariant. It was predicted that an interaction between Skemp's scores and the two labs would take place. From the literature it would seem reasonable to predict that students with low scores in Skemp's test would achieve better in the directed laboratory since more directions are given. Among the student with high Skemp scores, achievement would be better in the non-directed laboratory since there was more opportunity to think reflectively.

CHAPTER IV

THE EXPERIMENTAL DESIGN

As stated in Chapter I, the purpose of this study was to attempt to relate student scores on two different cognitive variables with achievement in two distinct types of mathematics laboratories. In particular, Kagan's categorical variable of impulsivity and reflectivity and Skemp's continuous variable of reflective intelligence were studied in relationship to achievement in the directed and non-directed laboratories. The experimental design used in the study is discussed in this chapter. The chapter includes a description of the instructional setting, the instructional and sampling procedures, the tests and instruments used in the experiment, the null hypothesis, and the statistical procedures used.

I. Instructional Setting

Eighty-seven students from four Grade 10 mathematics classrooms took part in the study. The students were first grouped into pairs according to friendship criteria and then these pairs were randomly assigned in a blind fashion to either of the two treatments. The two treatments, the directed and non-directed laboratories, had parallel activities although they differed in instructional presentation. The students were told that the laboratories were different but the difference was not in the amount of work or the kind of activity but rather in

the order of presentation.

Each group of students was given a show box containing equipment needed for the lessons and a lesson booklet. The lesson booklets investigated six mathematical concepts, the concepts of counting outcomes, permutations, combinations, probability, independent events and dependent events. These concepts were based on the chapter on probability from the Mathematics 15 textbook used in the Edmonton, Alberta Public Schools. Typical laboratory activities included counting, arranging, charting and estimating.

The booklet was divided into four section and each of these sections were printed on different colored paper to enable the students to quickly find the particular section they were studying and to enable the teacher to see at a glance whether a student was falling behind the rest of the class.

Johnson and Rising (1967) have suggested that one of the characteristics of the low achiever is a high rate of absenteeism. For this reason, topics within each section were not sequenced; that is, the last lesson in a section could be done before the preceding ones without the student having any difficulty in understanding the lessons. This lack of sequencing thus kept the students who were periodically absent from falling behind the rest of the class.

Since the students worked in groups of two, they were able to exchange ideas, guesses and observations. The students dependence on the teacher for information was also minimized since difficulties arising in the understanding of the concepts could be discussed freely amongst the students. The teacher's main function was general super-

vision and individualized consultation. It should be noted that the teacher needed to be as active as the students. The lessons were not designed to provide programmed instruction; therefore, the students would often require help from the teacher. It was the teacher's responsibility to act as a guide for the students who were having difficulty and to make sure that the students were working.

II. The Choice Of Subject Matter

The subject matter used in the mathematics laboratories was a unit on probability based on a chapter from the Math 15 textbook. Although Math 15 students are weak in basic arithmetic and algebraic skills, repetition of these skills tends to bore them, therefore, the material in the experiment tried to disguise the practice of these skills and at the same time teaches new concepts about probability.

Probability is an important field for the average man as well as an exciting one. As the Life Science Library book on Mathematics says in the chapter on "The Fascinating Game of Probability and Chance":

> " For one brief moment when a coin is tossed into the air, it assumes a state of unpredictability. No one can say which face will come up. Yet, toss that coin a million times and it will, with increasing minor variations, come heads half the time and tails the rest. In essence, this is the basis of the theory of probability - a branch of mathematics which deals with likelihoods, predictabilities and chance. First enuciated 300 years ago, probability's earliest applications were in the field of gambling to which it still has very strong ties. But probability (like its handmaiden, statistics) has become an indispensible modern tool predicting everything from life expectancies - in humans and light bulbs alike - to positions of electrons. The Frenchman Pierre Simon de LaPlace, preeminent in the field of probability, called it a science which began with play but evolved into 'the most important object of human knowledge'". (1963, P. 134).

III. The Sample

The sample consisted of 87 low achieving students from four different grade 10 non-academic mathematics classrooms. Of the 87 students that participated in the experiment, 63 students completed all of the tests. These students were taking Math 15 because they had failed the previous years work and they were required to take at least one mathematics course in order to obtain a high school diploma. The I.Q. sample exhibited a range of 86 to 134 and a reflective intelligence range of 0 to 31, clearly illustrating a heterogeneity of the group on these two variables.

Three high schools in the Edmonton Public School System were used in the study. More than one school was used to enable better representation of the general population. For example, one school might contain pupils from only one socio-economic status. For this reason, two of the classrooms were from Victoria Composite High School, a city center school, and there was one classroom from Strathcona Composite High School and one from Queen Elizabeth Composite High School which are located away from the city center.

These schools used 80 minute daily instructional periods, rather than the shorter 40 minute periods found in most high schools in the Province of Alberta. This additional time is the result of the use of the semester system. Instead of teaching one subject over the duration of an entire school term, the semester system divides the school term into two equal parts, the courses being completed at the end of each half. This system is becoming increasingly more popular in Alberta. Since low

achievers are often considered difficult to teach, many teachers are finding difficulty teaching the full 80 minutes. Materials are needed to help the teacher teach these longer periods. Therefore, in this study the semester system with 80 minute periods was chosen.

IV. The Procedure

A. Pilot Study

Difficulty invariably arises when new curriculum materials are developed. Ambiguities and poorly worded phrases often lead the student to misunderstand the concepts. For this reason the materials used in the study were first piloted at M.E. Lazerte High School in Edmonton. After difficulty arose in the scheduling of the mathematics laboratories at M.E. Lazarete, the investigator piloted the materials with twelve students from Victoria Composite High School.

The pilot study at Victoria Composite High School also helped the investigator develop the teachers guide and the suggested timetable. The results from the pilot study suggested that this method of teaching is appropriate for low achievers. On an achievement test given at the end of the unit, the students marks ranged from 50% to 87% with an average of 68%. This was a bit surprising since many of the achievement test questions were based on Grade 12 government departmental examinations. This pilot study also helped to establish the reliability and validity of the achievement test.

An attitude test was given to the students. It seemed to indicate that the students enjoyed the unit on probability. Also attendence was remarkably improved although one girl did drop out of school after

attending only two lessons. A factor affecting the apparent rise in both achievement and attitude may have been due to the freedom given to the students. The student were allowed to talk and mix freely, thereby exchanging ideas as often as desired.

B. Training Program

Prior to the beginning of the experimental study the students did an introductory lesson on probability. This lesson acted as a training program for the students and teachers who were unfamiliar with the mathematics laboratory approach.

C. The Schedule

The following minimal time units were suggested. The teachers were told to take more time if they felt that they needed it, but not to use less than the suggested time. Since the materials had been piloted in Math 15 classes, these times generally seemed to be appropriate minimums for successful completion.

TABLE 1

SCHEDULE FOR LABORATORY PROGRAM

No. of Days	Section	Name of Concept
1	0	Introduction
2	1	Counting
2-3	2	Ordering and Combination
2-3	3	Probability
3-4	4	Independent & Dependent Events

V. Tests And Instruments

The following sections give a brief summary of the tests used in the study. Skemp's reflective intelligence test was administered by the classroom teacher before the experimental program was started. It took approximately eighty minutes to administer. The achievement test was given at the conclusion of the unit of mathematics laboratories. It was administered by the classroom teacher and took the students approximately a full classroom period. Kagan's Matching Familiar Figures Test was administered individually at the conclusion of the laboratory unit, under the supervision of the investigators and two testers trained by the investigator. The Lorge-Thorndike I.Q. test scores were obtained from the school records.

A. Kagan's Matching Familiar Figures Test

Kagan (1963) developed the MFF (Matching Familiar Figures Test) to measure individual differences in the speed of solving a problem. The adult version of this test was used in the study. In the test, the tester shows the subject a picture of a familiar object such as a leaf. The subject is then asked to choose from eight variants a picture which is identical to the standards (see Appendix E for a sample of Kagan's test). The investigator records to the nearest half second the time which it takes the subject to decide his first response along with the number of errors that were made in answering each of the twelve questions correctly.

According to Kagan's research the number of errors a person makes is inversely proportional to the time it takes the person to respond to

the questions. The data used in this study substantiated this fact. The Pearson-product moment correlation between the number of errors made in responding to the questions and time it took to respond to each question was -0.600 which is significant at the .01 level. Kagan has described the subject who makes rapid nonanalytic responses as impulsive and the subject who makes delayed analytic responses as reflective. Statistically that is, those subjects in the study above the median of 12 on the total number of errors and below the median of 400 seconds total time for all 12 questions were classified as impulsive thinkers and those subjects below the median on the number of errors and above the median on the time scale were classified as reflective thinkers. The remaining subjects were left unclassified as they did not fit into either definition of relfectivity or impulsivity. (see Appendix E for the M.F.F. test).

The following table gives the number of students in each category in the two sample laboratory setting.

TABLE 2

NUMBER OF STUDENTS IN EACH CATEGORY

Type of Laboratory	No. of Reflective Students	No. of Impulsive Students	No. of Unclassified Students	Total
Directed	15	9	11	35
Non-Directed	11	12	5	28
Total	26	21	16	63

B. Skemp's Reflective Intelligence Test

Skemp (1958) developed a two part test to measure reflective intelligence (see Chapter II and III for Skemp's definition of reflective intelligence). The test used in this study was titled SK6 according to Harrison's (1967) classification. SK6 was designed by Skemp to measure the student's ability to perform operations (Part I) and the ability to manipulate these operations (Part II). Part I consisted of a problem sheet and a demonstration sheet. The demonstration sheet has three examples of simple abstract line figures representing fifteen operations. The subject was asked to examine these operations and then perform the operations on three figures for each question on the problem sheet. An example of a typical operation is a clockwise rotation through a right angle or a reflection in a horizontal line. After the subjects had completed the test their papers were collected and the answers to the questions were discussed. The mean average of the SK6 Part I was 26.8 out of 30, therefore, it could be concluded that the students generally understood Part I.

The second part of the test was designed to measure the ability of the student to manipulate the operations. Part II has fifteen questions. The first five questions involved the combination of two operations, one after another, the second five questions involved reversing a single operation and the third five questions involved first combining and then reversing the operations (see Appendix G for SK6 test).

C. Lorge-Thorndike I.Q. Test

Lorge-Thorndike and Mufflin (1954) developed an I.Q. test. The test consisted of two parts, the verbal battery and the non-verbal battery. The verbal battery is a subtest of verbal items which act as a good index for scholastic aptitude. The non-verbal battery of a subtest of pictorial or numerical items. Although the non-verbal battery does not predict achievement as well as the verbal test, it does give an estimate of scholastic aptitude which was not influenced by reading ability. Since the verbal and non-verbal scores correlate highly (approximately .70) the ordinary differences between the scores will not be significant for the majority of pupils (Cronbach 1960).

D. Achievement Test

The achievement test consisted of 30 multiple choice questions, each question having four choices. Many of the questions were taken from the twelfth grade Alberta Departmental exams. The questions dealt with the five concepts developed in the mathematics laboratory (see Appendix D for the achievement test).

The mean average of the achievement test was 15.50 out of 30 as compared to the results of 20.4 in the pilot study. The reliability as computed by the KR-20 method was 0.6718. The following formula is used to compute the KR-20 reliability:

$$r_{xx} = \frac{n}{n-1} \quad \frac{SX^2 - \sum_{t=1}^{n} p_i \, q_i}{S_x^2}$$

where n is the number of test items

S_x^2 is the variance of the test score as defined as $\frac{(X-X)^2}{n}$

p_i is 1 for the position of individuals passing the item "i"

$q_i = 1-p$ or the portion of individuals failing the item "i"; that is,

p_iq_i is the product of the portion of passes and fails for each item, therefore, $\sum_{i=1}^{n} p_iq_i$ is the sum of products for n items.

The validity of the test was substantiated by the pilot study.

VI. The Variables

The following is a list of the variables used in answering the null hypothesis. The independent variables were:

1. Kagan's impulsive and reflective variables.
2. Skemp's reflective intelligence variable.
3. Lorge-Thorndike's I.Q. verbal and non-verbal I.Q. variable.

The dependent variable was:

4. The mathematics achievement variable.

VII. The Null Hypotheses

A. Major Hypothesis

Two major hypotheses were developed in this study and a number of minor hypotheses were derived from the major hypotheses. The two major hypotheses were:

Null Hypothesis I

There is no significant interaction between Kagan's variable and

the treatments (the directed and the non-directed laboratories) when the measure used as a dependent variable is the achievement scores and when the Lorge-Thorndike I.Q. scores are used as covariates.

Null Hypothesis II

There is no significant interaction between Skemp's variable and the treatments when the measure used as a dependent variable is the achievement score and when the Lorge-Thorndike I.Q. scores are used as covariates.

B. Covariates

As the result of null hypotheses I and II the interaction with the Lorge-Thorndike I.Q. scores needed to be tested to see if the I.Q. scores could be used as covariates. To test for covariates the following null hypotheses were tested.

Null Hypothesis III

There is no significant interaction between Lorge-Thorndikes non-verbal I.Q. variables and the treatments when the measure used as a dependent variable is the achievement score.

Null Hypothesis IV

There is no significant interaction between Lorge-Thorndike's verbal I.Q. variable and the treatments when the measure used as a dependent variable is the achievement score.

C. Main Effects

The following hypotheses test indicate whether the variables created any main effects. The testing of these hypotheses is contingent upon whether there is an interaction in Hypothesis 1 and 2.

Null Hypothesis V

There is no significant difference between the mean achievement scores of the student in the directed and non-directed laboratories.

Null Hypothesis VI

There is no significant difference between the mean achievement scores of the impulsive and reflective learner.

Null Hypothesis VII

There is no significant correlation between Skemp's reflective intelligence scores and the achievement scores.

D. Replication

The following hypotheses were tested to try to substantiate Skemp's (1958) and Harrison's (1967) claim that reflective intelligence measures a variable that is completely separate from what the I.Q. Variable measures.

Null Hypothesis VIII

There is no significant correlation between Skemp's reflective intelligence scores and the Lorge-Thorndike verbal I.Q. score.

Null Hypothesis IX

There is no significant correlation between Skemp's reflective intelligence score and the Lorge-Thorndike non-verbal I.Q. score.

VIII. The Statistical Procedure

In preparation for data processing on the University of Alberta's digital computer, an IBM data card containing the test scores and an ID number was punched for each student in the study who had written all of

the tests that had been administered. The statistical procedure followed in analysing the data from the study is described in the following paragraphs. A list of the scores obtained by the 63 students who had complete sets of data is included in Appendix H.

There were two types of variables used in the study - the continuous variable and the categorical variable. A continuous variable is a variable composed of scores with an assumed underlying distribution in which any score is possible and the units are equal in size (Flathman, 1968). Examples of the continuous variables used in the study were Skemp's reflective intelligence score and Lorge-Thorndike's non-verbal I.Q. score. In a categorical variable,the subject belongs to one and only one group. In the case of Kagan's MFF score, the students were classified as impulsive, reflective or left unclassified.

The statistical procedure used in making decisions with respect to Null Hypcthesis I and II was based on multiple regre;sion-interaction analysis. In this approach a functional relationship was assumed between the dependent variable (Y) and the independent variables (X_i) is Y = f (X). This relationship is assumed to be linear and additive that is, the model would have the following form:

$$Y = A_oU + A_1X_1 + A_2X_2 + AX + 33 \ldots. + A_nX_n + E$$

where Y is the dependent (criterion variable)

$X_1, X_2, \ldots, X_n$ are the independent (predictor) variables

U is a unit vector

A_o is a constant for all subjects

$A_1, A_2 \ldots A_n$ are regression weights or regression coefficients,and

E is the error term which allows for the possibility that it might not be possible to predict Y.

If all of the X's are independent the full or unrestricted model has N + 1 degrees of freedom (degrees of freedom are the number of independent predictors in the model). To investigate the independent contribution to one of these variables, the restricted model incorporates the null hypothesis that a particular X_1 makes no contribution. For example, to test whether X_1 makes any independent contribution, one lets $A_1 = 0$.

$Y = A_oU + A_1X_1 + A_2X_2 + \ldots + A_nX_n$ would be the full model.

$Y = A_o'U' + A_2'X_2' + A_3'X_3' + \ldots A_n'X_n'$ would be the restricted model.

An F-test is used to compare the squared multiple-correlation between these two models. If F is sufficiently large, the null hypothesis is rejected and the conclusion that X_1 does make an independent contribution is reached. The value of the F test is computed by the following formula:

$$F = \frac{(R_1^{\ 2} - R_2^{\ 2}) \ / \ (df_1 - df_2)}{(1 - R_1^{\ 2}) \ / \ (N - df_1)}$$

where subscripts 1 and 2 refer respectively to the restricted and unrestricted models

df stands for degrees of freedom

R^2 (the squared - multiple correlation) is the square of the correlation between the observed and the predicted criterion scores.

In the numerator the F ratio has $(df_1 - df_2)$ degrees of freedom and in the denominator the F ratio has $(N - df_1)$ degrees of freedom where N is the size of the population in the sample (Flathman, 1968).

In the study, Null Hypothesis I and II contain variables which are generated from two of the predictor vectors. A variable which is the

product of two other variables represents an interaction variable between the two variables. For example, suppose X_1 is the predictor vector for Skemp's scores and X_2 and X_3 are respectively predictor vector for the directed and non-directed laboratories. X_4, X_5 could be defined in the following manner

$$X_4 = X_1X_2$$

$$X_5 = X_1X_3$$

thus, X_4 and X_5 represent the interaction between these two types of laboratories and Skemp's test. By including the interaction terms in the model, the possibility that one variable depends upon the value of the other variable is taken into account. Without the interaction terms, the two variables are assumed to have an independent effect.

Null Hypotheses I, II, III and IV were tested by using interaction regression models while Null Hypothesis V and VI used ordinary multiple regression models. Null Hypothesis VII, VIII and IX were tested by Pearson-product Moment Correlations. The Division of Educational Research Services in the University of Alberta have two prepared programs to test these hypotheses*.

* The investigator received help from Dave Flathman to write the regression models (Program MULR 05) and help was received from Ernest Skakun for writing the Pearson-product Moment Correlations (Program DESTO 5).

CHAPTER V

RESULTS OF THE STUDY

The purpose of the study was to investigate the relationship between two types of mathematics laboratories and two cognitive variables. This chapter reports the statistical results pertaining to the hypothesis of the study. The findings are reported under the headings corresponding to the null hypothesis discussed in Chapter IV. In presenting the findings to each question, the null hypothesis is re-stated, the testing procedure is described and the results of the analysis are given.

I. Major Hypotheses

A. Hypothesis I

Re-stated: There is no significant interaction between Kagan's variable and the treatments (the directed and the non-directed laboratories) when the measures used as a dependent variable is the achievement score and when the Lorge-Thorndike I.Q. Score is used as a covariant.

Test Procedure: A multiple regression model was used to test the interaction between Kagan's variable and the two treatments. See model 01 and model 02 in Table 4 for the full and restricted models used to test this hypothesis.

Results: Null Hypothesis I was not rejected. No significant interaction between Kagan's variable and the laboratory treatments was found. The F ratio for Hypothesis I was 0.3496. This F ratio has a

probability of 0.70647 for 2/55 degrees of freedom. See Table 4 for the regression weights of models 01 and 02, and see Table 6 for the mean averages of the different groups.

B. Hypothesis II

Re-stated: There is no significant interaction between Skemp's variable and the treatments when the measure used as a dependent variable is the achievement score and when the Lorge-Thorndike I.Q. score is used as a covariate.

Test-Procedure: A multiple regression model was used to test the interaction between Skemp's variable and the two treatments. See Model 03 and 04 in Table 4 for the full restricted models used to test this hypothesis.

Results: No significant interaction effects were observed in null hypothesis II; that is, there is no significant interaction between Skemp's scores and the achievement scores of the two laboratory treatments. The F ratio for Hypothesis II was 0.0001. This F ratio has a probability of 0.99337 for 1/57 degrees of freedom. See Table 4 for the regression weights of Models 03 and 04.

II. Covariates

A. Hypothesis III

Re-stated: There is no significant interaction between Lorge-Thorndike's non-verbal I.Q. variable and the treatments when the measure used as a dependent variable is the achievement score.

Test Procedures: A multiple regression model was used to test the interaction between Lorge-Thorndike's non-verbal I.Q. variable and the treatments. See Model 05 and 06 in Table 4 for the full and restricted models used to test this hypothesis.

Results: No significant interaction effects were observed in Null Hypothesis III, therefore non-verbal I.Q. scores can be used as a covariate. The F ratio for Hypothesis III was 0.5573. This F ratio has a probability of 0.458226 for 1/59 degrees of freedom. See Table 4 for the regression weights of Models 05 and 06.

B. Hypothesis IV

Restated: There is no significant interaction between Lorge-Thorndike's verbal I.Q. variable and the treatments when the measure used as a dependent variable is the achievement score.

Test Procedure: A multiple regression model was used to test the interaction between Lorge-Thorndike's verbal I.Q. variable and the treatments. See Model 07 and 08 in Table 4 for the full and restricted models used to test this hypothesis.

Results: No significant interaction effects were observed in null hypothesis IV, therefore, verbal I.Q. scores can be used as a covariate. The F ratio for Hypothesis IV was 0.1548. This F ratio has a probability of 0.69534 for 1/59 degrees of freedom. See Table 4 for the regression weights of models 07 and 08.

III. Main Effects

A. Hypothesis V

Re-stated: There is no significant difference between the mean achievement scores in the directed and non-directed laboratories.

Test Procedure: A multiple regression model was used to test the main effects between the mean achievement scores of the two treatments. See Model 09 and 10 in Table II for the full and restricted models used to test this hypothesis.

Results: No significant main effect was observed in Null Hypothesis V, that is, there is no significant difference between the mean achievement scores in the two treatments. The F ratio was 0.0117. The probability of this F ratio is 0.91434 for 1/61 degrees of freedom. See Table 4 for the regression weights of model 09 and see Table 5 for the means of the directed and non-directed laboratories.

B. Hypothesis VI

Re-stated: There is no significant difference between the mean achievement scores of the impulsive and reflective learner.

Test Procedure: A multiple regression model was used to test the main effect of the mean achievement scores of the impulsive and reflective learner. See Model 10 and 99 in Table 4 for the full and restricted models used to test this hypothesis.

Results: No significant main effect was observed in Null Hypothesis VI; that is, there is no significant difference between the mean achievement scores of the impulsive and reflective learner.

The F ratio was 1.3012. The probability of this F ratio is 0.25843 for 1/61 degrees of freedom. See Table 5 for the achievement means of the two categories.

C. Hypothesis VII

Re-stated: There is no significant correlation between Skemp's reflective intelligence scores and the achievement scores.

Test Procedures: The Pearson-product moment correlation between Skemp's score and the achievement score was tested at the .05 level.

Results: The correlation of .396 between Skemp's scores and the achievement scores is significant at the .05 level and at the .01 level.

IV. Replication

A. Hypothesis VIII

Re-stated: There is no significant correlation between Skemp's reflective intelligence scores and the Lorge-Thorndike verbal I.Q. score.

Test-Procedure: The Pearson-product moment correlation between Skemp's score and the verbal I.Q. score was tested at the .05 level.

Results: The hypothesis was not rejected; that is, the correlation of the 0.245 between Skemp's scores and the verbal I.Q. scores was not significant at the 0.05 level.

B. Hypothesis IX

Re-stated: There is no significant correlation between Skemp's reflective intelligence scores and the Lorge-Thorndike non-verbal I.Q. scores.

Test-Procedure: The Pearson-product moment correlation between Skemp's scores and the non-verbal I.Q. score was tested at the 0.05 level.

Results: The hypothesis was rejected; that is, there is a significant correlation of 0.280 between Skemp's reflective intelligence scores and the Lorge-Thorndike non-verbal I.Q. scores at the .05 level.

TABLE 3

Definitions of the Variables

Variable No.	Name of Variable
1	Skemp's SK6 score
2	Kagan's MFF number of errors score
3	Kagan's MFF time score
4	Achievement score
5	Verbal I.Q. score
6	Non-Verbal I.Q. score
7	= 1 Directed Laboratory, 0 otherwise
8	= 1 Non-directed laboratory, 0 otherwise
9	= 1 Reflective personality, 0 otherwise
10	= 1 Impulsive personality, 0 otherwise
11	= 1 unclassified Kagan's category, 0 otherwise
12	= X_1X_7 Directed Skemp scores
13	= X_2X_8 Non-directed Skemp scores
14	= X_7X_9 =1 if directed & reflective individual,0 otherwise
15	= X_7X_{10} =1 if directed & impulsive individual, 0 otherwise
16	= X_7X_{11}=1 if directed & unclassified individual, 0 otherwise
17	= X_8X_9=1 if non-directed & reflective individual, 0 otherwise
18	= X_8X_{10}=1 if non-directed & impulsive individual, 0 otherwise
19	= X_8X_{11}=1 if non-directed & unclassified individual, 0 otherwise
20	= X_5X_7 directed verbal I.Q. score
21	= X_5X_8 non-directed verbal I.Q. score
22	= X_6X_7 directed non-verbal I.Q. score
23	= X_6X_8 non-directed non-verbal I.Q. score

TABLE 4

PREDICTION EQUATIONS & WEIGHTS

Model 01

$$Y_5 = A_oU + A_5X_5 + A_6X_6 + A_7X_7 + A_9X_9 + A_{10}X_{10} + A_{14}X_{14} + A_{15}X_{15} + A_{17}X_{17} + A_{18}X_{18} + E$$

RSQ = 0.14468652

Variable No.	Regression Weight
5	0.05602007
6	0.08182573
7	0.76711658
9	-0.57137748
10	-0.87468668
14	-0.55516530
15	-0.28421748
17	1.58809663
18	0.33692887
Constant	0.25046492

Model 02

$$Y_5 = A_oU + A_5X_5 + A_6X_6 + A_7X_7 + A_9X_9 + A_{10}X_{10} + E$$

RSQ = 0.13381308

Variable No.	Regression Weight
5	0.05952754
6	0.09011279
7	-0.35923868
9	-0.35717974
10	-1.08918236
Constant	-0.27185488

Model 03

$$Y_5 = A_oU + A_5X_5 + A_6X_6 + A_7X_7 + A_{12}X_{12} + A_{13}X_{13} + E$$

RSQ = 0.24723756

Variable No.	Regression Weight
5	0.08190966
6	0.04831057
7	-0.63428064
12	0.18714692
13	0.18621041
Constant	-1.15086365

Model 04

$$Y_5 = A_oU + A_1X_1 + A_5X_5 + A_6X_6 + A_7X_7 + E$$

RSQ = 0.24723673

Variable No.	Regression Weight
1	0.18677223
5	0.08189264
6	0.04830739
7	-0.62331189
Constant	-1.15468502

Model 05

$$Y_5 = A_oU + A_7X_7 + A_{22}X_{22} + A_{23}X_{23} + E$$

RSQ = 0.10569930

Variable No.	Regression Weight
7	6.83026261
22	6.08069507
23	0.14467730
Constant	-0.37407577

Model 06

$$Y_5 = A_oU + A_6X_6 + A_7X_7 + E$$

RSQ = 0.09725183

Variable No.	Regression Weight
6	0.11020368
7	-0.26069931
Constant	3.42541313

Model 07

$$Y_5 = A_oU + A_7X_7 + A_{20}X_{20} + A_{21}X_{21} + E$$

RSQ = 0.06233254

Variable No.	Regression Weight
7	4.05382573
20	0.07680611
21	0.11394582
Constant	3.09477139

Model 08

$$Y_5 = A_oU + A_5X_5 + A_7X_7 + E$$

RSQ = 0.05987215

Variable No.	Regression Weight
5	0.09118113
7	0.00681556
Constant	5.58711910

Model 09

$$Y_5 = A_oU + A_7X_7 + E$$

RSQ = 0.00019100

Variable No.	Regression Weight
7	-0.11428572
Constant	15.57142830

Model 10

$$Y_5 = A_oU + A_9X_9 + A_{10}X_{10} + E$$

RSQ = 0.02088505

Variable No.	Regression Weight
9	-0.25113128
10	-1.4088264
Constant	16.05882263

TABLE 5

MEAN AVERAGES

Test Title	Means
Skemp Reflective Intelligence	14.68
Kagan Error	12.14
Kagan Total Time	521.61
Achievement	15.51
Verbal IQ	108.76
Non-Verbal IQ	110.95
Achievement in Directed Lab	15.45
Achievement in Non-Directed Lab	15.57
Achievement for Impulsive Students	14.95
Achievement for Reflective Students	15.81

TABLE 6

MEAN AVERAGES OF HYPOTHESIS I

	Directed	Non-Directed	Total Average
Reflective	14.86	17.09	15.80
Impulsive	15.22	14.75	14.95
Total Average	15.00	15.86	

TABLE 7

THE PEARSON-PRODUCT MOMENT CORRELATION RESULTS

	Skemp Scores	Kagan No. of Errors	Kagan Time	Achie-vement Score	Verbal I.Q. Score	Non Ver-bal I.Q. Score
Skemp's Scores	1.000	-0.072	-0.080	0.396	-0.048	0.307
Kagan No. of Errors	-0.072	1.000	-0.600	0.009	-0.102	-0.057
Kagan Time	-0.080	-0.600	1.000	-0.118	0.203	-0.056
Achievement Score	0.396	0.009	-0.118	1.000	0.245	0.310
Verbal I.Q.	-0.048	-0.102	0.203	0.245	1.000	0.280
Non-verbal I.Q.	0.307	-0.057	-0.056	0.310	0.280	1.000

TABLE 8

PROBABILITY THAT R=0 FOR THE CORRELATIONS

	Skemp Scores	Kagan No. of Errors	Kagan Time	Achie-vement Score	Verbal I.Q. Score	Non Ver-bal I.Q. Score
Skemp's Scores		0.575	0.533	0.001	0.712	0.014
Kagan no. of Errors	0.575		0.000	0.944	0.425	0.655
Kagan Time	0.533	0.000		0.355	0.111	0.665
Achievement Score	0.001	0.944	0.355	0.0	0.053	0.013
Verbal I.Q.	0.712	0.425	0.111	0.053	0.0	0.026
Non-Verbal I.Q.	0.014	0.655	0.665	0.013	0.026	0.0

CHAPTER VI

CONCLUSIONS AND IMPLICATIONS

I. Purpose and Design of the Study

The major purpose of the study was to investigate the relationship between two cognitive learning styles and two types of mathematics laboratories. Two mathematics laboratory programs comprised of eighteen lessons were developed around a unit on probability from the mathematics 15 program taught in Alberta. Mathematics 15 is a non-academic program offered as credit for a high school diploma to all students who had failed the previous years work in mathematics. The study involved classes drawn from three high schools in Edmonton using a semester system. In a semester system the length of the school term for each subject is cut in half meaning that the class periods are increased from 40 minutes to 80 minutes a day.

The students were grouped into pairs according to a friendship criteria and then randomly assigned to either of the two treatments. The two treatments, the directed and non-directed laboratories, had parallel activities but differed in instructional technique. The difference between the programs was that the directed laboratory had an advance organizer in the form of a set of additional questions placed at the beginning of the lessons (see Chapter 2, Part I for the definition of advance organizers). The program of instruction took three to four weeks to complete, depending on the class.

Each group was given a laboratory booklet and a shoe box containing the equipment needed for the laboratory lesson. Prior to starting the program, the students were given a sample lesson to help familiarize them with the laboratory method of instruction. It was the teacher's responsibility to make sure that the students were given appropriate guidance. Since some students are slower than others, some of the lessons were made optional, therefore, not all of the students completed all of the lessons, but all completed the core lessons.

As independent variable measures, Skemp's test of reflective intelligence was administered to classroom groups and Kagan's Matching Familiar Figures Test was administered individually to each of the participating students. As a dependent variable, a measure of achievement in probability was administered directly at the end of the three week treatment period.

II. Summary of Results

Analysis of the data obtained during the investigation revealed the following results:

1. There was no significant interaction between Kagan's variable and the achievement scores of the two treatments when the Lorge-Thorndike IQ scores were used as covariates.

2. There was no significant interaction between Skemp's variable and the achievement scores of the two treatments when the Lorge-Thorndike IQ scores were used as covariates.

3. There was no significant interaction between Lorge-Thorndike's non-verbal IQ scores and the achievement scores of the two treatments.

4. There was no significant interaction between Lorge-Thorndike's verbal IQ scores and the achievement scores of the two treatments.

5. There was no significant differences between the mean achievement scores in the directed and non-directed laboratories.

6. There was no significant differences between the mean achievement scores of the impulsive and reflective learner.

7. There is significant correlation between the achievement scores and Skemp's reflective intelligence.

8. There was no significant correlation between Skemp's scores and the verbal IQ scores.

9. There was a significant correlation between Skemp's scores and the non-verbal IQ scores.

III. Conclusions and Discussions

In the study, there were two major hypothesis - one that tested the interaction between Skemp's reflective intelligence score and the achievement score in two types of mathematics laboratories and the other that tested the interaction between Kagan's impulsive and reflective variable and the achievement scores. The data collected in the study did not substantiate these hypotheses. The following section discusses this and other results of the study.

Analysis related to hypothesis II analyzed the interaction between Skemp's reflective intelligence variable and the achievement variable

when IQ was used as a covariate. From the review of the literature related to Skemp's reflective intelligence and the two types of laboratories, the following interaction was expected. It was predicted that the students who obtained low Skemp scores would learn better when using the directed laboratory program and the students who obtained high Skemp scores would learn better when using the non-directed laboratory program. Since the results showed that there was no statistical interaction between Skemp's variable and the achievement variable, these two variables were tested for main effects.

Analysis related to hypothesis VII tested the strength of the relationship between Skemp's variable and the achievement variable. As would be expected from the literature related to Skemp's theory, reflective intelligence did correlate significantly with the achievement score. Thus the data relating to Hypothesis II and VII suggested that reflective intelligence relates in a similar positive way to achievement in both laboratory settings. This result further contributes to the conclusion of Skemp (1958) and Harrison (1967) that reflective intelligence is a good predictor of mathematics achievement. This study verified this conclusion for low achieving mathematics students in a non-conventional classroom setting. Harrison (1967) hypothesized that reflective intelligence score as measured by Skemp's test was separate from or different from IQ scores. Since Lorge-Thorndike IQ scores were available for the students in the study, hypothesis VIII and IX were designed to test Skemp's variable with the non-verbal and verbal variables of Lorge-Thorndike's test. The correlation between Skemp's variable and the verbal IQ score was found to be non-significant while

the correlation between Skemp's variable and the non-verbal IQ score was found to be significant. Since Skemp's test is a symbolic test, it is not surprising that non-verbal IQ scores correlate positively (0.307). The correlation between the verbal IQ scores and Skemp's score was almost non-existent and slightly negative (-0.048). Therefore, it can be concluded that verbal IQ and Skemp's reflective intelligence are not closely related and that Skemp's score is a better predictor of mathematic's achievement for the students in this study.*

Since Skemp's test proved to be significantly related to achievement, it is possible that the reason no significant interaction took place in hypothesis II was because there was no significant difference between the two types of laboratories. Hypothesis V was designed to test the significant difference between mean achievement in the two types of laboratories. The results showed that there were no significant difference between the two laboratories in achievement. From a review of literature on which the design of the two types of laboratories were developed, it might be assumed that students in the directed laboratory should have a higher mean achievement than the non-directed laboratory (Ausubel, 1963). The mean achievement of students in the non-directed laboratory was 15.57 while the mean of students in the directed laboratory was 15.45 (not in the direction of the prediction). Therefore, the conclusion might be made that hypothesis V did not result in significant difference because there was not a big enough difference in the instructional methodology between the two laboratory programs, or that differential impact of the two methods did not affect achievements.

* Skemp correlated with achievement at 0.397, non-verbal IQ correlated at 0.310 and verbal at 0.245.

Hypothesis I tested the interaction between Kagan's variable of impulsive and reflective personalities and the achievement in the two mathematics laboratories. From a review of the literature it was predicted that impulsive students would do better in the directed laboratory lessons because the lesson gave more direction and that the reflective students would do better in the non-directed laboratory lessons. The results of the study show the direction of the predictions were correct although not significant (see Table 6 in Chapter 5 for the means). Since the results were not significant, the main effects were tested.

Hypothesis VI was designed to test the significant difference in achievement between the impulsive and reflective students. According to the research done on Kagan's variable the reflective students should have been significantly better achievers than the impulsive students. Although the statistical results did not substantiate these results, the results did point in the right direction; that is, reflective students had a mean of 15.61 while the impulsive students had a lower mean of 14.95. Therefore, Kagan's variable did not discriminate for the low achieving sample used in the study. The lack of significant differences between the achievement in the two groups might be explained by the specialized sample. Since the students were low achievers(and according to Kagan's theory of conceptual tempo) it would be reasonable to predict that most of the sample would be impulsive students. In fact, the opposite took place when Kagan's method of median split on time and error scales was used to divide the students into two groups. This method resulted in 26 reflective students and 21 impulsive students out of a sample of 63. Perhaps, the median split method of dividing the groups might not have

been appropriate for the low achieving sample used in this study.

In both hypothesis I and II the IQ scores were used as covariates; that is, the IQ score was felt to be a factor which should be taken into account. As a result hypotheses III and IV were designed to test if IQ score had any interaction effects on the achievement of the two laboratories. The statistical results showed that there was no significant interaction between either the verbal or non-verbal IQ scores and the achievement in the two types of laboratories; therefore, IQ could be used as a covariate.

Another compounding variable was the teacher variable. Table 9 below shows the breakdown of the classrooms according to the central and suburban schools. It should be noted that both suburban schools had slightly higher IQ levels than the central school classrooms and that central school classroom A had much lower results on all of the tests. Thus, a teacher or class setting effect may have confounded the results of the study.

TABLE 9

SUMMARY OF CLASSROOM AVERAGES

School	No. of Students	Verbal IQ	Non-Verbal IQ.	Achievement
Suburban A	24	108.66	112.04	15.29
Suburban B	15	110.80	114.46	17.93
Central A	16	104.93	106.68	12.50
Central B	8	112.87	109.62	17.87

IV. Implications for Further Research

This section discusses several ways this study could be expanded or improved upon. The first section discusses different possible instructional techniques that could be developed into laboratory lessons. The second section discusses different variables which could be used in an aptitude interaction study.

A. Differences between Laboratories

There are many avenues to explore in developing different instructional techniques for mathematics laboratories. As facilities become better and materials become more plentiful, selection of materials best suited for a particular individual's needs will become a more common happening. In the case of the study, the distinction between the two types of laboratories was developed around the idea of the advance organizer as described by Ausubel. These two laboratories did not result in any significant difference in achievement with the particular sample used.

There are many other instructional methods for mathematics laboratories which could have been chosen. For example, different types of lessons could be written around problems which distinguish between the rule and the solutions are given. Shulman (1970) has described four types of instructional techniques which involve rule-solution problems. Problems in which the rules and solutions are given are described as expository problems; ones in which only the rules are given are described as guided discovery (deductive) problems; ones in which only the solutions are given are described as guided discovery (inductive) problems; and ones

in which neither the rules or solutions are given are described as "pure" discovery lessons.

Another possibility for developing different types of laboratory instruction is to write laboratories which differ in inductive and deductive reasoning. The deductive technique is similar to an expository strategy which was defined by Eldredge (1965) as rule-example learning. On the other hand, the inductive method of reasoning, from specifics to generalities, is very similar to the discovery method of hint-then-formula. Although Ausubel has agreed that the discovery process is less efficient than the expository process, he points out that self-discovery has more incentive for the child (Shulman, 1970).

Another technique could be defined by the differences between the laboratories in terms of divergent and convergent thinking. Guilford (1959) has suggested that the distinctive aspect of a creative thinker is divergent thinking, which is characterized by flexibility, originality and fluency. This type of thinking produces many hypotheses to a single correct answer. Gallagher (1964) showed that a teacher can determine how the student thinks by the type of questions he asks. For example:

> " In teaching Hamlet the teacher may ask, 'Explain why Hamlet rejected Ophelia'. The student may go through this convergent thinking process in answering this question: Hamlet became disillusioned with all women. Ophelia was a woman. Hamlet rejected Ophelia. The teacher, however, could stimulate divergent thinking by asking these questions: What other ways could Hamlet have used to trap the king? What other courses or action might have been open to Hamlet's mother? Suppose Polonius had not been killed; how would that have modified or changed the eventual outcome of the play?" (De Cecco, 1968, P. 455).

Therefore, depending on how the questions in the laboratory lessons were stated, the student could be taught in a convergent or divergent manner.

B. Variables

Skemp's reflective intelligence variable seems to be a strong predictor variable for achievement even in a group of non-academic students. Therefore it would appear that Skemp's variable would make a good covariate. A further study on whether the growth of this reflective capacity can be increased by various methods of instruction might prove valuable.

There are four main entering characteristics that a student brings with him when starting a new unit of work. These characteristics are readiness, personality type, general attitude and cognitive style. Readiness is a variable which takes into consideration the adequacy of the student's existing capacity. It includes prior knowledge one has of the subject matter and the maturation or developmental level of the student. Maturation refers to the biological growth which occurs largely under the influence of heredity. In maturation, certain structural changes must occur before a certain type of behavior can appear. (This agrees with Piaget's developmental theory.)

The psychology of personality is largely the study of hypothetical structures and processes within a person that cause him to act in a certain way (De Cecco, 1968). Undoubtedly, many entering behaviors are a result of different personality structures. Variables such as introversion, anxiety or verbal non-verbal orientation might lead to assigning certain types of students to special types of instruction. But it has been pointed out by Richard Lindeman (1967) the usefulness of having teachers employ personality test is dubious. He suggests:

> " There are at least two reasons for this.
> 1. The teacher usually has his hands full if he is to do a satisfactory job in areas of intellectual development and, 2. investigations into student personality characteristics usually require competence and training distinctly different from those possessed by the classroom teacher... the use of instruments for assessing personality by persons without proper training may result in real harm to the child (De Cecco, 1968, P. 676).

Attitudes and values are another category of variables which could be explored in relation to different instructional techniques. For a further discussion on attitude variables see the accompanying thesis to this study written by John Odynski.

Finally different cognitive styles could be useful variables for distinguishing which type of student benefits most from a particular type of instructional technique. As suggested in the preceding section, convergent and divergent thinking could be a useful method of distinguishing between two types of laboratories. Equally as useful would be to use a measure of convergence and divergence as a variable to study in relationship to achievement in different instructional setting. Convergent thinking produces a single correct answer, whereas divergent thinking produces a variety of responses. One special aptitude test is the tests of creativity designed by Guilford. His tests are based on his hypothesis that creative thinking is divergent thinking. He feels that creative or divergent thinking is the result of three factors: flexibility, originality and fluency. He has developed several tests to measure these factors. For example the Hidden Figures and the Match Problem Tests measures flexibility, the Plot Title Test, the Symbol Test and the Production Test measures originality and the Brick Uses Test measures fluency.

Taylor-Pearce concentrated on divergent production as defined by Guilford, and developed tests measuring students ability to produce many answers to a question about concepts in Grade 10 mathematics. He scored the responses for fluency, variety, novelty, the latter being refinements of Guilford's flexibility, originality measures. The students ability to produce responses in a divergent manner may influence his behavior in a non-directed laboratory. A student who likes to produce many hypotheses may relate more appropriately in a non-directive situation.

Another creativity test that might be useful is Flanagan's Ingenuity Test. In this test the student must think of ingenious and effective ways of doing things (De Cecco, 1968). Torrance has also written a series of tests on creative thinking. He defined creativity as a process of becoming sensitive to problems by finding their gaps in knowledge, identifying difficulties, searching for solutions and testing and retesting problems. He has developed five verbal tests- Ask-and-guess activity, Product improvement, Unusual uses, Unusual questions and Just Suppose - and three non-verbal tests - Picture construction, Picture completion and Parallel lines.

Another promising set of variables is Jerome Bruner's (1956) selection strategies. These variables describe the various ways an individual can learn concepts through the order in which examples and non-examples of a concept appear. Four selection strategies have been identified. In the first, conservative focusing, the individual uses an example of the concept as a focus and changes one attribute of the example at a

time to find which attributes are essential. This strategy almost guarantees success. Focus gambling is a selection strategy in which the individual focuses on an example but he changes more than one attribute at a time. This strategy allows for the possibility of acquiring the concept much quicker. It is similar to Kagan's conceptual tempo where impulsive children use focus gambling while reflective children use conservative focusing. In the third strategy, simulatenous scanning, the student makes several hypothesis about the attribute of the concept. Each time he encounters a different idea which incorporates the concept, he decides to keep or eliminate some of the hypothesis. In the final strategy, successive scanning, the student tests a single hypothesis at a time. Individuals may vary their strategy with the concept or they may retain a particular strategy regardless of the concept they seek to acquire. Low-risk strategies such as successive scanning and conservative focusing may cause students to be reflective. In any case, selection strategies are one means of classifying entering behavior (De Cecco, 1968); therefore this variable might be a useful predictor of the type of student who will do well in a particular type of laboratory lesson.

One final variable which might prove fruitful is Diene's (1965) thinking structures. Diene's classifies different levels of thought into three categories - memory, pattern and operation thinking. In memory strategy the individual's learning strategy is random since each solution is memorized independently. Next in the hierarchy is the pattern strategy where an individual searches for a general pattern or table on which to place solutions and, finally, the operational strategy is one where the individual presupposes that part of the pattern acts on (or

is a function of) one other part. Diene's has developed an instrument to measure this strategy. In the study an attempt was made to study eight students using Diene's test. Unfortunately, the tester was unable to have a single individual complete the test. Thus, classification of the sample was impossible (see Appendix I for a fuller account of Dienes' variables).

The above variables describe some of the possible variables which could be studied in an aptitude-treatment interaction study. Further research into the relation of these variables with achievement in various instruction setting will some day help us answer the question concerning attitude-treatment interaction.

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APPENDIX A

SOURCES AND REFERENCES FOR THE LABORATORY ACTIVITIES AND CONCRETE MATERIALS

SOURCES AND REFERENCES FOR THE LABORATORY
ACTIVITIES AND CONCRETE MATERIALS

Section			
Introduction	Sample Lab-Probability	School mathematics Study Group, Introduction to Probability, Patt 1, 1966	1 ordinary die 1 "instant insanity" block
1. Counting Outcome	A. Pick your Choice	Developed by Linda Kreklewetz	2 different colored dice; a pink and blue gameboard
	B. In Jail	Activities in Mathematics First Course "Probability" Scott Foresman & Co., 1971	Pencil; Drawing of puzzle
	C. Chain Letter	Developed by Katherine McLeod	Chips of 2 different colors
	D. Pascal Triangle	Experience in Mathematics Discovery-Arrangements & Selection, Vol. 5, National Council of Teachers of Mathematics, 1967	4 different coins
2. Combinations & Permutations	A. The Downhill Racer	Developed by Katherine McLeod	Chips of 4 different colors
	B. Jim's Dilemma	Developed by John Odynski	chips of 4 different colors
	C. Blind Man	Kimmey, L.B., Ruble, V. & Brown, G.W. General Mathematics, Book One Holt, inehart & Winston of Canada Ltd., 1969	1 bag; 8 white chips; 4 black chips; 4 red chips
	D. Football Seats	Developed by John Odynski	100 cards: 25 with the name Ann, 25 with the name Liz, 25 with the name Penny, 25 with the name Susan
	E. Station 13	Developed by Katherine McLeod	chips of 3 different colors; cards of 2 different colors

	F. Circular Arrangements	Developed by John Odynski	chips of 5 different colors
3.Probability	A.Grashopper Game	Activities in Mathematics First Course "Probability" Scott Foresman & Co., 1971	die gameboard
	B.Football Game	Developed by John Odynski & Dr.T.Kieren	3 different coins
	C. Bag It	Developed by John Odynski	20 white chips 20 blue chips a bag
4.Independent & Dependent Events	A.Die vs. Coin	Developed by John Odynski & Dr.Olson	1 die 1 coin
	B.What Color?	Developed by John Odynski & Dr.Olson	5 white chips 5 blue chips a bag
	C.Red,Orange, or Blue,Green Yellow	Developed by John Odynski	10 red cards; 2 green cards, 6 blue cards; 2 yellow cards; 5 orange cards; a bag
	D.6 or Under, 7 or Over	Developed by John Odynski	2 different colored dice

APPENDIX B

SAMPLE LABORATORY LESSONS FOR BOTH THE DIRECTED AND NON-DIRECTED LABORATORY

A. DIRECTED LABORATORY

PICK YOUR CHOICE

Objective: counting outcomes of events.

Materials: 2 different colored dice, and a pink and blue gameboard.

1. When throwing two dice do you have a better chance of winning with a sum of 6, 7 or 8 than with a sum of 2, 3, 4, 5, 9, 10, 11, 12? Why?

2. Perhaps the following table will help you answer the above question. Complete the table. The first number stands for the outcome on the green die and the second number stands for the outcome on the red die.

TABLE A

Die 2 (green) \ Die 1 (red)	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Remember that an outcome of 2 on the green die and 5 on the red die (we'll write it as (2,5)) is different from an outcome of 5 on the green die and 2 on the red die (we'll write this as (5,2)).

(a) How many ways can you throw a sum of 8? List them.

(b) How many ways can you throw a sum of 7? List them.

(c) How many ways can you throw a sum of 6? List them.

(d) Answer #1 again.

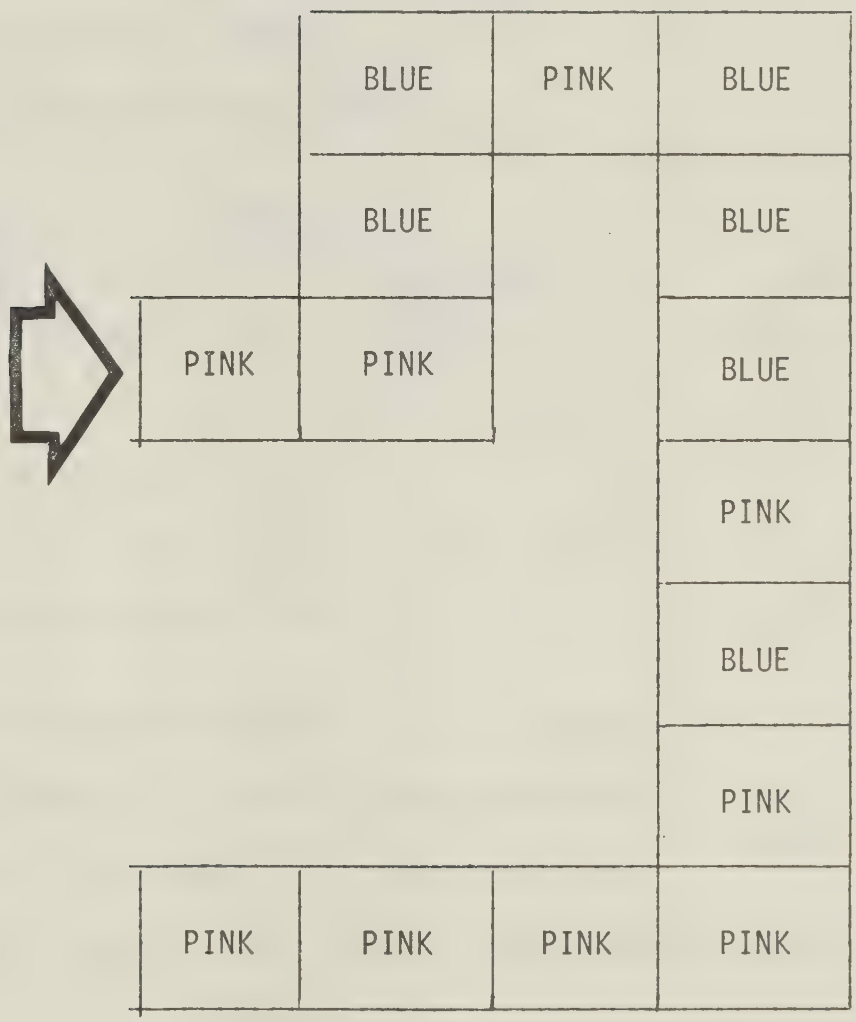
Two players are to play this game.

1. There are 15 squares, some are blue and some are pink on large cardboard.

2. Flip a coin. The winner chooses the color he prefers and will toss the 2 dice while the other person fills in Table 1.

GAMEBOARD

START for each toss of 2 DICE



3. Toss 2 dice. Make the same number of moves in the diagram as the sum of the dots on the two dice.

4. If you end on a pink - give pink one point.
If you end on a blue - give blue one point.

5. On each toss of the dice, always start counting at "start".

6. Toss the dice 20 times (1 game).

7. Repeat this 4 times (or play 4 games).

TABLE 1

	Tally for Pink	Tally for Blue	Score Pink Blue
Game 1			
Game 2			
Game 3			
Game 4			

1. Do you think this game is fair? Why?

2. (a) How many squares are there on the gameboard?

   (b) Do you need this many? Why or why not?

3. Complete Table 2 below. Hint: Use the table you made on the first page.

TABLE 2

Sum of Dice	List Possibilities	No. of outcomes
1	Two dice can't add up to 1	0
2	(1, 1)	1
3		
4	(1,3) (2,2) (3,1)	3
5		
6		
7		
8		
9		
10		
11		
12		

4. Refer to Table 2.

   (a) In tossing two dice, which sum has the BEST chance of appearing?

   (b) In tossing two dice, which 2 sums have the LEAST chance of appearing?

5. (a) Will a sum of 7 land you on the blue?

   (b) Will a sum of 2 or 12 land you on pink?

6. How many different ways can you throw two dice?

7. Out of the 36 possible outcomes of throwing 2 dice how many of these will yield a point for the blue?

8. How many outcomes will yield a point for pink?

   There are twice as many possible outcomes for the blue as there are for the pink.

9. Again, is this a fair game? Explain.

B. NON-DIRECTED LABORATORY

<u>Pick Your Choice</u>

Objective: counting outcomes of events

Materials: 2 different colored dice and a pink and blue gameboard

Two players are to play this game.

1. There are 15 squares, some are blue and some are pink on a large cardboard.

2. Flip a coin. The winner chooses the color he prefers and will toss the 2 dice while the other person fills in Table 1.

3. Toss 2 dice. Make the same number of moves in the diagram as the sum of the dots on the two dice.

4. If you end on a pink - give pink one point.
   If you end on a blue - give blue one point.

5. On each toss of the dice, always start counting at "start".

6. Toss the dice 20 times (1 game).

7. Repeat this 4 times (or play 4 games).

TABLE 1

	Tally for Pink	Tally for Blue	Score	
			Pink	Blue
Game 1				
Game 2				
Game 3				
Game 4				

Is this game fair? Explain.

1. (a) What is the highest number you can throw with two dice?

   (b) The lowest?

2. How many squares are there on the gameboard?

3. (a) Do you need that many? How many do you need?

   (b) Which ones can you land on when you throw the dice?

4. When throwing two dice do you have a better chance of winning with a sum of 4 or 7? Explain.

5. Perhaps the following table will help you answer the above question. Complete Table 2.

TABLE 2

		Die 1 (red)					
		1	2	3	4	5	6
	1						
	2						
Die 2 (green)	3						
	4						
	5						
	6						

6. (a) Will a sum of 7 land you on the blue?

   (b) Will a sum of 12 or 2 land you on pink?

7. How many different ways can you throw two dice?

8. Out of the 36 possible outcomes of throwing 2 dice, which of these will yield a point for blue? Shade in Table 2.

9. Which outcomes will yield a point for pink?

10. Again, is this a fair game? Explain.

A. DIRECTED LABORATORY

The Downhill Racer

Objective: combinations and permutations

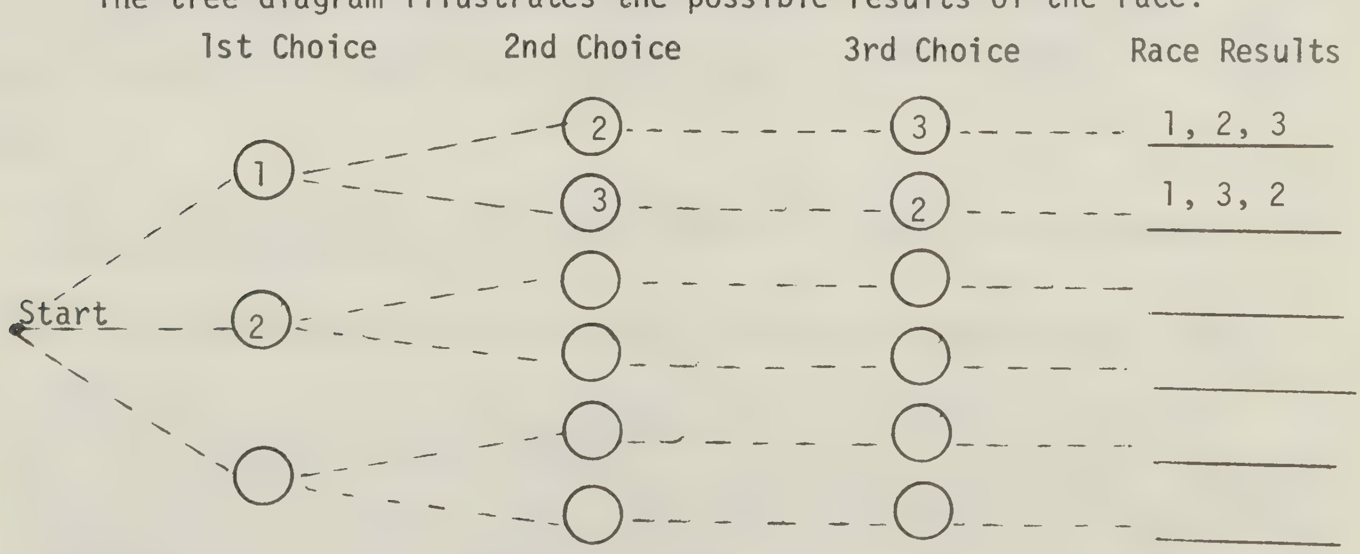
Materials: chips of 4 different colors or chips of 3 different colors and cards.

1. If you have three cars in a car rally
   (a) How many cards could finish in first position?

   (b) After first place has been won, how many cars could come second?

   (c) How many cards could come third after the first two positions have been won?

2. If the cars are number one to three, complete the tree diagram below. The tree diagram illustrates the possible results of the race.



3. (a) You have how many choices for 1st position?

   (b) After 1st position is chosen, you have how many choices for 2nd position?

   (c) For third position?

4. The total number of orders you can have are ______.

5. What can you do with the answers in question 3 to get the answer for question 4? In other words what mathematical operations (+,-,x,÷) can you do with 3, 2, 1 to get 6. (make two rules)

   The following game is designed to help you test your rules so that you might discover which rule is correct.

   THE REST OF THE LESSON IS THE SAME AS THE NON-DIRECTED LABORATORY.

B. NON-DIRECTED LABORATORY

<u>The Downhill Racer</u>

Objective: combinations and permutations

Materials: Chips of 4 different colors or chips of 3 different colors plus cards.

In ski races, the competitors do not run the race at the same time but start at regular intervals one after the other. Why?

Often it is advantageous to be the first person to compete in a race because as more skiers race down the hill the ruts tend to slow your speed. The following game is designed to illustrate the possible <u>orders</u> or <u>arrangements</u> in which four skiers might begin a race.

<u>Rules</u>:

1. There are two players. Each player takes his turn after the other player has completed his turn.

2. The 4 different colored chips stand for the four different ski racers.

3. Decide who is to be the first player. The first player places four different colored chips in any order on the table (e.g. R, G, B, Y). The color which has first position or starting position remains the same during the first players turn. The first player tries to make as many <u>arrangements</u> or starting <u>orders</u> for the race as possible by interchanging second, third and fourth place skiers. Use new chips for each arrangement so that you can check to see that no two arrangements are alike.

1 point is awarded for each different order that you make. 2 points are lost for every mistake your opponent finds at the end of your turn. You can make mistakes by

(a) putting down the same arrangement twice.

(b) by forgetting to make a certain arrangement.

4. Continue this procedure until all the skiers have had a chance to begin the race in first position.

5. DO NOT DESTROY ANY OF YOUR PREVIOUS ARRANGEMENTS.

Answer the following questions:

1. How many different ways can the race be run?

2. How many choices of skiers do you have for first position?

3. After you have chosen first position, how many skiers remain to be placed into second place?

4. In third position, you would have how many choices?

5. How many choices for 4th position do you have after 1st, 2nd, and 3rd position have been chosen?

6. What mathematical operation (+, -, x, ÷) can you do with the answers to questions 2, 3, 4, 5 to get the answer of question 1?

7. Make a mathematical rule.

8. Check to see if your rule works with

   (a) 5 skiers

   (b) Did you get 120 different ways of racing by multiplying 5 X 4 X 3 X 2 X 1?

   (c) Now try for 3 skiers.

A. DIRECTED LABORATORY

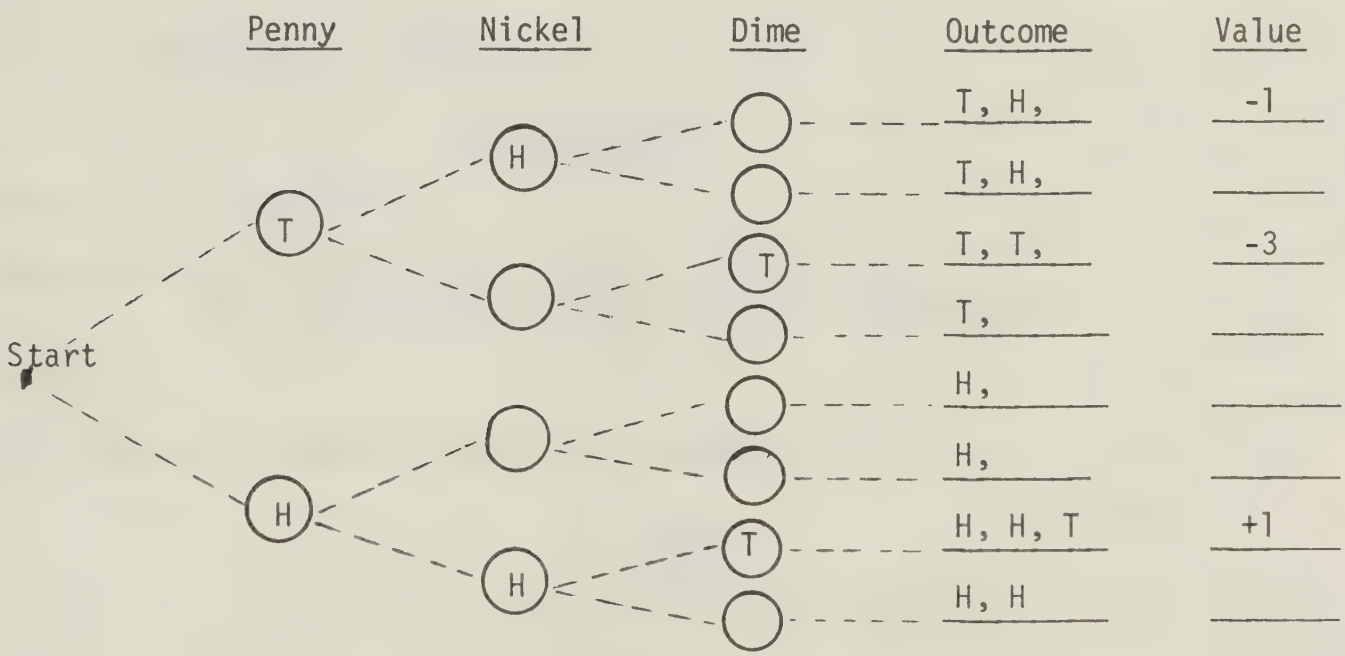
Football Game

Objective: Probability

Materials: 3 different coins, i.e. penny, nickel, dime.

1. If one tossed three coins, there are various ways in which the coins may land. If for each tail that landed a value of -1 was assigned, and for each head that landed a value of +1 was assigned, in how many ways by summing the values of the 3 coins can you get +1? -1? +2? -2? +3? -3? 0?

   If you are having problems complete the following tree diagram:



2. In the case above you can never get the values -2, +2 and 0. Why not? These are called impossible events. What type of situations would you need to get -2, +2, and 0 if you were tossing 4 coins?

3. In tossing 3 coins there are 8 different possible ways of them landing. Calculate the following ratios:

   (a) ratio of getting a total -1 out of the 8 possible ways

   (b) ratio of getting a total of +1 out of the 8 possible ways

   (c) ratio of getting a total +3 out of the 8 possible ways

(d) ratio of getting a total of +2 out of the 8 possible ways

(e) ratio of getting a total of 0 out of the 8 possible ways

(f) ratio of getting a total of -3 out of the 8 possible ways

Total ________

Check: The ratios for (a) and (b) should be the same.
The ratios for (c) and (f) should be the same.
The ratios for (d) and (3) should be the same,

= 0/8 = 0.

All the 8 ratios should add up to 1.

Note: The ratio for any impossible event is 0

B. NON-DIRECTED LABORATORY

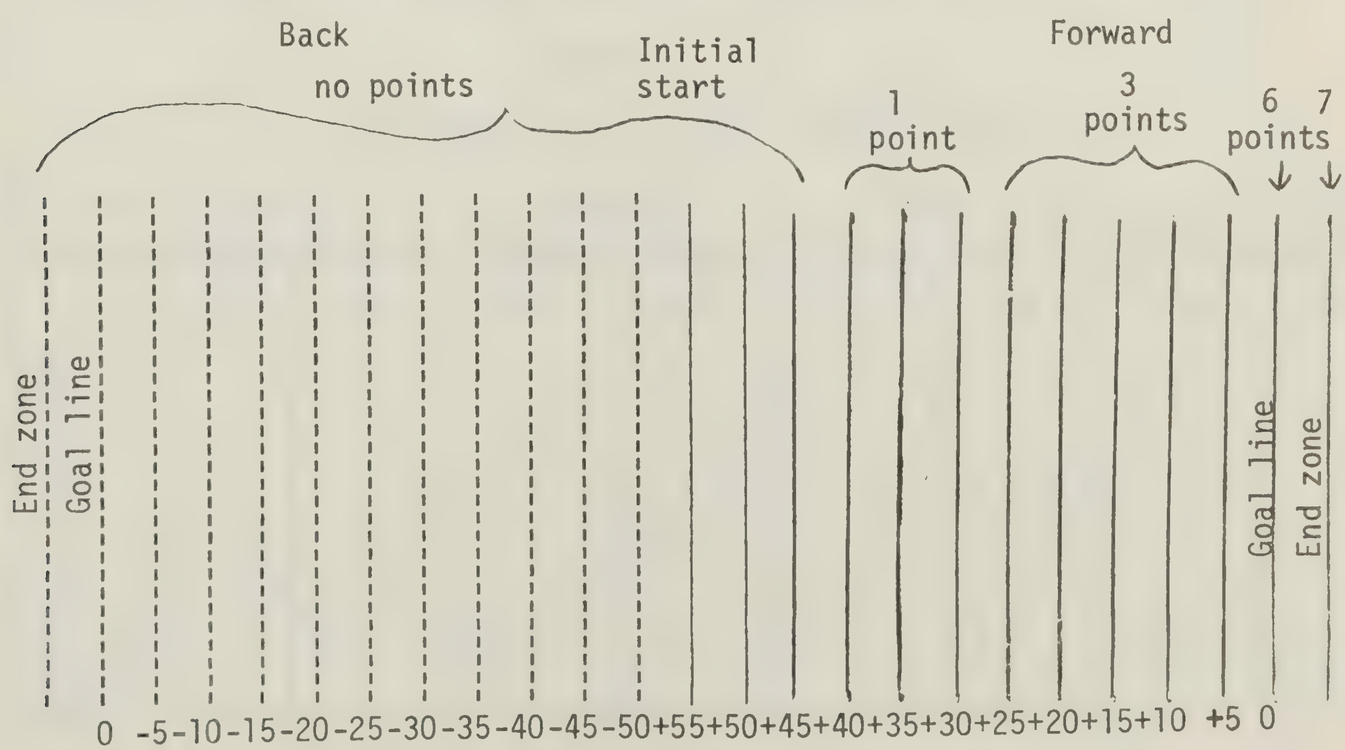
Football Game

Objective: Probability

Materials: 3 different coins; penny, nickel, dime.
One or two people are to play the following game.

Rules:

1. Below is a diagram of part of a football field.



2. Flip a coin, the winner starts and we'll call him the fullback. The other person will be the fumbler.

3. When a person starts to toss the 3 coins, he is on the +55 yard line.

4. One person is to toss the coins ten times, then the other person will toss the coins 10 times (This makes one game). Each person starts at 55 yardline.''

5. For every head that lands, move 5 yards forward, for every tail move 5 yards back. For example, if coins land T H H your moves would be +55 to -50 to +55 to +50 (you would record +50 in Table 1). If on next toss the coins land H T H your moves would be (your position now is at +50) +50 to +45 to +50 to +45. (Now record +45 in Table 1).

6. At the end of 10 tosses, if your position is

   (a) between 40 and 30 yard lines inclusive ________1 point

   (b) between 25 and 5 yard lines inclusive ________3 points

   (c) on the goal line ____________________6 points

   (d) over the goal line or in end zone __________7 points

   NOTE: If you got over the goal line before 10 tosses you have 7 points, return to +55 yard line and continue tossing the coins until you have tossed them a total of 10 times.

7. Play four games - your original names as in game one will remain the same for sake of filling out Table 1.

8. Keep track of position and score for each group in Table 1. The person who is not tossing the coins should fill in the table.

TABLE 1

Yardline at end of each toss of 3 coins

No. of Tosses	Game 1		Game 2		Game 3		Game 4	
	Fullback	Fumbler	Fullback	Fumbler	Fullback	Fumbler	Fullback	Fumbler
0	+55	+55	+55	+55	+55	+55	+55	+55
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
Score								

1. Notice that during each game most of your positions on the football field are usually not beyond the +35 yardline. You may even have not reached the +35 yard line. Have you any explanation for this? (HINT: Note the different ways the three coins may land and list them.)

2. Which of the eight possible ways the coins may land will produce a move in a forward direction? These favorable cases are what fraction of the total number of ways the coins may land?

3. The ratio you have calculated in question 2 (which should have been 1/2) is called probability.

$$\text{Probability} = \frac{\text{number of favorable outcomes}}{\text{total no. of possible outcomes}}$$

   Calculate the probability for each of the different ways the three coins may land.

   The eight probabilities you have calculated should be all the same and all of them add up to 1.

4. What is the probability of any two of the three coins landing heads?

   Is this the same as the penny, nickel and dime landing T H H respectively? Why not?

5. Do you think this game was fair? Why?-

6. If an event or outcome could never occur such as in tossing 3 coins and you wanted to get 3 heads and 1 tail in the same toss, what would you call such an event?

A. DIRECTED LABORATORY

<u>Red, Orange, or Blue, Green, Yellow</u>

Objective: Dependent and Independent Events

Jim sat on a street corner and observed 100 cars passing by. 10 of these were Volkswagons, 25 were Chevs, 30 were Fords, 20 were Pontiacs and 15 were Toyotas.

1. What is the probability of a Volkswagon passing by?

2. Is the probability of a Pontiac 3 times that of the probability of a Volkswagon? If yes, check your answers for questions 1 and 2.

3. Jim now wondered what the probability was for a General Motors Card (Chev or Pontiac) to pass by.

   (a) The probability of a Chev is ______

(b) The probability of a Pontiac is 20/100 = ______

(c) If one is calculating the probability of the next car being a Chev or a Pontiac, certainly the next car CANNOT be BOTH a Chev and a Pontiac. It can ONLY be a Chev OR a Pontiac. Therefore add the 2 probabilities. That is 25/100 + 20/100 = 45/100 = ________

4. Another way of calculating the probability for a Chev or a Pontiac is as follows:

$$\text{Probability} = \frac{\text{total number of favorable events}}{\text{total number of possible events}}$$

Since there is a total of 45 Chevs or Pontiacs, then the probability of the next car being a Chev or a Pontiac is

$$\frac{\_\_\_}{100} = \frac{9}{20}$$

B. NON-DIRECTED LABORATORY

Red, Orange or Blue, Green, Yellow

Materials: 10 red cards, 2 green cards, 6 blue cards, 2 yellow cards, 5 orange cards, bag.

Two people are to play the following game.

Rules:

1. Flip a coin. The winner chooses the route he prefers to follow in scoring a point. The other person gets the other route.

2. There are two ways of scoring a point.

   (a) Picking a red or orange card.
   (b) Picking a blue, green or yellow card.

3. The winner is to pick a card and notice the color. If he chooses the correct card he scores a point. Return the card to the bag and mix the cards. Pick again, notice the color and score a point if applicable. Return the card and mix them. Repeat this 20 times. Now let the other person pick a card 20 times.

4. One game is played when each person has had 20 picks.

5. Play 4 games.

6. Fill in Table 1.

TABLE 1

	GREEN, BLUE, YELLOW			RED, ORANGE			WINNER	
	TALLY			TALLY				
	Successful	Not Successful	Score	Successful	Not Successful	Score	Green Blue Yellow	Red Orange
Game 1								
Game 2								
Game 3								
Game 4								

1. Which one was the most successful (red-orange or green-blue-yellow)?

2. Is this game fair? Explain.

3. Let's take a look at red-orange scoring a point. There is a total of 25 cards of which 10 are red. The probability of choosing a red is____. The probability of chossing an orange is______. Since you can ONLY choose a red OR an orange card on one pick, the two probabilities are added. In other words, the probability for choosing a red or an orange card is 10/25 + 5/25 = 15/25 = _____.

4. Another way of calculating the probability for choosing a red OR orange card is as follows. There are 10 red and 5 orange cards. Therefore there are 15 favorable picks our of a total of 25. The probability then of choosing a red OR orange card is:

$$\frac{\text{total number of favorable choices}}{\text{total number of possible choices}} = \_\_\_\_\_\_\_\_$$

5. Let's take a look at green-blue-yellow scoring a point.

   (a) There are 6 blue cards. The probability of choosing a blue is ___.

   (b) There are 2 green cards. The probability of picking a green one is _______.

   (c) The probability of picking a yellow one is ________.

   (d) Since you can ONLY pick one of these favorable colors at a time, will you (add, subtract, multiply or divide) the probabilities?

   (e) The probability of picking a blue or a green or a yellow card is:

$$\frac{6}{25} + \frac{2}{25} + \frac{2}{25} \doteq \frac{10}{25} = \frac{2}{5}$$

6. Looking at probability as being

$$\frac{\text{total number of favorable choices}}{\text{total number of possible choices}}$$

then the total number of favorable choices (that is the total number of green, blue and yellow cards) is:____. The probability of choosing a green, blue or yellow card is _______. Did you get 10/25 = 2/5 again? If not, check your work.

7. Again is this game fair? Explain.

APPENDIX C

TEACHER'S GUIDE

TEACHER's MANUAL

RATIONALE

In the last decade educational research in the field of mathematics has mainly focused attention on the average and above average, college-bound student. The low achiever is only now beginning to be recognized. For this reason, we have designed a unit on probability in conjunction with the Math 15 program. This unit emphasizes the following principles.

1. CONCRETE MATERIALS - Several well-known psychologists such as Piaget and Bruner have suggested that learning new concepts in its initial stages can best be done with the aid of concrete materials; therefore, all the lessons in the unit will allow the student to manipulate concrete materials.

2. SOCIAL PARTICIPATION - The students will work in groups of two or three since interaction between individuals can help speed up the process of concept attainment.

3. MULTIPLE EMBODIMENT - This principle implies that each concept be taught in several ways. For this reason there are several laboratory lessons for each of the concepts.

4. SHORT LESSONS - The lessons are deliberately kept short to help maintain the interest of the students. An attempt is made to use everyday language. However, you should expect to help students interpret in a few places and students should expect to read what there is to read.

5. INDIVIDUALIZATION - Since students work at different rates, optional lessons have been written for the faster students. Therefore,

NOT ALL of the students will do ALL of the lessons.

6. DISCOVERY AND NATURE OF THE LESSONS - The lessons are designed to allow the student to discover the mathematics concept for himself as much as possible.

Two types of laboratory lessons have been developed - the directed and non-directed lab. The essential difference between these two labs is that in the directed lab the students are given some mathematical background about the particular lesson before playing the game or trying the problem of that particular laboratory lesson. In the non-directed laboratory lessons, the students play the game or try the problem first and then they try to extract the mathematics behind the lab.

PROCEDURE

1. Do NOT be confused by the names directed and non-directed laboratory. The way in which you act in both lab settings should be the SAME.
2. Please explain to the students that both labs contain equal amounts of work but are just written in a different manner.
3. Group the students in pairs according to friendship criteria. NOTE: It is important that one member of the group can read.
4. There are two types of lab booklets. Randomly assign half of the group to one type of lab and other other half to the other type. (Please make sure that all the "best" students don't get one type of lab.)
5. Assign each group one booklet. Each booklet contains the written instructions for about 2 - 3 weeks of work.
6. The students are required to complete all the lessons except those which

are optional. The optional lessons are provided for those students who work faster than the rest of the group. They provide further reinforcement of the concepts that are to be learned.

7. There are 5 basic concepts in the booklet. Each concept is printed on a different color of paper so that the student and teacher can easily find the section they are on.

8. Because the booklet is short term, this method need not be a true individualized approach. The teacher might wish to conduct a class discussion on topics that the students are having problems with.

9. You might expect more noise in the classroom because the students will be working in pairs. However, this has not proven to be a problem previously if the students are working.

10. NOTE: These activity-oriented labs require that the <u>teacher be active</u>. The lessons are NOT programmed learning so that students will need to be taught parts of the lesson. The teacher is to act as a guide to the pupils and NOT to leave them on their own during the duration of the lab lessons.

11. The class should take up the questions at the end of each section. (The questions are on the white pages.) Here the teacher might use a more conventional method of teaching.

12. Students can check their answers for the lab lessons from the teacher's manual if they wish.

HOW TO INTRODUCE THE MATERIAL

Teacher

1. For the initial introduction demonstration ask, "How many different ways do you (the class) think a deck of cards can be shuffled?" After a discussion where the students guesses are recorded they might be interested in the following information. "The total of different sequences possible in a 52-card deck is a figure 68 numerals long; if all the people on the earth counted a million arrangements a second 24 hours a day for 80 years, they could not count a billionth of a billionth of 1 per cent of the possibilities. The total number of 5 card poker hands possible is 2,598,960. The number of 13-card bridge hands: 635,013,559,600". (page 139, Life Science Library - Mathematics).

2. Read with the students the introduction to probability.
   (a) The question on page 2 is a brainstorming question to build up some enthusiasm. See who can put the most occupations down in 3 minutes and then discuss.
   (b) Then continue reading and do the newspaper questions. Try to get the students to talk freely.

3. Explain to the students about how they are going to be working in pairs for the next two to three weeks and that each pair is going to have one booklet which cannot be taken from the classroom.

TIMETABLE SUGGESTIONS

1. Attitude test (approximately 10-15 minutes). Only designated pupils write. Pupils to write their names on these tests but explain that in no way will their results on this test affect their marks. Student's are to be honest in their feelings toward Math 15.

2. Skemp's test - until finished (approximately 30 minutes).

3. Start on Introduction to Probability. (See Introduction sheet.)

* The following time limits are suggested. Please feel free to take more time if needed, but do not use less than the suggested minimum time. The materials have been piloted with Math 15 classes. These times generally seem to be appropriate minimums for <u>successful</u> completion.

No. of Days	Section	Name of Concept
1	0	Introduction
2	1	Counting
2 - 3	2	Ordering and Combination
2 - 3	3	Probability
3 - 4	4	Independent & Dependent events

* When finished with the labs.

1. Attitude test for everyone (10 - 15 minutes). Give attitude test the day <u>before</u> the achievement test.

2. Achievement test - devote a full period to this test.

APPENDIX D

MULTIPLE CHOICE ACHIEVEMENT TEST

TEST - COUNTING, ORDERING, COMBINATIONS & PROBABILITY

Circle Your Choice

1. Set A contains 7 elements. The total number of possible orders by taking 4 elements at a time is:

   A) 7 X 6 X 5 X 4   C) 4 X 4 X 4 X 4

   B) 7 X 6 X 5 X 4 X 3 X 2 X 1   d) 7 X 7 X 7 X 7

2. The Jones couple wishes to name their son so that his initials are ACJ. If they choose from Alvin, Alexander, Andrew, Alfonse, Chuck, Curtis, Colin, in how many ways might the son be named?

   A) 1   B) 3   C) 4   D) 12

3. In how many ways can 5 different colored marbles be arranged in a row?

   A) 5   B) 3125   C) 120   D) 15

4. A penny is tossed 3 times. Find the probability of obtaining 2 heads and 1 tail.

   A) 1/8   B) 3/8   C) 1/4   D) 1/2

5. A bag contains 2 white, 4 blue and 6 red marbles. If 1 marble is drawn from the bag, what is the probability that it is NOT blue?

   A) 2/3   B) 1/3   C) 1/8   D) 1/4

6. There are 4 redheads in a group of 30 people. The probability of selecting at random 2 redheads from the group is:

   A) $\frac{4}{30} + \frac{3}{29}$ B) $1 - \frac{2(26)}{2(30)}$ C) $\frac{4}{30} \text{ X } \frac{2}{26}$ D) $\frac{4}{30} \text{ X } \frac{3}{29}$

7. Two dice - 1 red, 1 green - are tossed. If the red die turns up 6, what is the probability of obtaining a total of 11 as the sum of the dots which show up on the two dice?

   A) 1/18   B) 1/2   C) 1/6   D) 2/3

8. There are 4 green, 3 white, and 5 red marbles in a bag. The probability that 2 marbles drawn (first marble is not replaced) from the bag will BOTH be white is:

   A) 1/11 B) 19/44 C) 1/24 D) 1/22

9. Find the probability that 2 heads will turn up when 2 coins are tossed:

   A) 3/4 B) 1/2 C) 1/4 D) 1

10. Three different colored dice are thrown at once. How many different outcomes are possible?

    A) 1 B) 216 C) 6 D) 36

11. In how many ways can 8 different books be arranged on a shelf?

    A) $\frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2}$ C) 8

    B) 7 X 6 X 5 X 4 X 3 X 2 X 1 D) 8 X 7 X 6 X 5 X 4 X 3 X 2 X 1

12. What is the probability of throwing two dice so that the sum of the dots facing up is nine?

    A) 1/9 B) 1/4 C) 1/3 D) 1/36

13.. How many different committees of 3 persons can be formed in a club having 6 members?

    A) 120 B) 20 C) 6 D) 720

14. A person throws 3 different coins at once. How many of the possible outcomes will contain exactly 2 heads?

    A) 8 B) 2 C) 3 D) 6

15. There are four different posters available for an arrangement in a row. The number of different arrangements that can be made if 2 posters are used is:

    A) 60 B) 24 C) 12 D)48

16. A die is rolled and a card is drawn from a deck of 52 playing cards. The probability of getting a 6 and a BLACK king is:

A) 1/6 x 2/52 B) 1/6 x 1/52 C) 1/6 x 2/51 D) 1/6 + 2/52

17. A card is drawn at random from an ordinary deck of 52 cards. What is the probability that the card is green?

A) 1/13 B) 1/4 C) 1/52 D) 0

18. A bag contains 3 black balls,2 white balls and 1 red ball. Two balls are drawn from the bag. If the first ball is replaced before the second ball is drawn, find the probability that both balls are black.

A) 1/2 B) 1/4 C) 1/9 D) 1/36

19. A team has 10 players. In how many ways can 2 players be selected as co-captains?

A) 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 C) 45

B) 90 D) 2

20. A regular hexahedron has 6 faces. One of its faces is painted white, 2 are painted red, and the rest are painted blue. If the hexahedron is rolled onto a flat surface, what is the probability that it will come to rest on a blue face or red face?

A) 5/6 B) 1/3 C) 1/2 D) 1/6

21. The probability of getting a sum of 8 on a toss of 2 dice is:

A) 1/22 B) 1/2 C) 1/12 D) 5/36

22. A town issues license plates for bicycles. Each plate has one of the 26 letters of the alphabet followed by one of the 10 digits. How many different plates are possible?

A) 26 B) 10 C) 26 x 10 D) 26 + 10

23. Three coins are tossed into the air. The probability that none lands heads up is:

A) 3/8 B) 1/8 C) 0 DO 1/2

24. For a family of three children, if each child is just as likely to be a boy as a girl, what is the probability that all three children are girls?

    A) 1/2 B) 1/4 C) 3/4 D) 1/8

25. All of the possible different outcomes for flipping a quarter and a penny is:

    A) (H,T)
    C) (H,H) (H,T) (T,T)
    B) (H,H) (H,T) (T,H) (T,T)
    D) (H,T) (T,H) (T,T)

26. A bag contains 6 red balls and 3 blue balls. If a die is thrown, what is the probability of getting 3 or less on the die <u>OR</u> picking a blue marble from the bag?

    A) 1/6 x 3/9 B) 3/6 x 3/9 C) 1/6 + 3/9 D) 3/6 + 3.9

27. A luncheon menu allows you to choose a soup from 4 different soups and a sandwich from 6 different sandwiches. How many different lunches can you have?

    A) 4 B) 6 C) 24 D) 10

28. All radio stations in Edmonton are named by 4 letters. The first letter is "C". How many radio stations can be obtained by filling in the blanks in "C---" with <u>three</u> letters to be chosen from the remaining 25 letters, without repetition?

    A) 13800 B) 26 C) 75 D) 15625

29. A card is drawn at random from an ordinary deck of 52 cards. What is the probability that the card is a heart?

    A) 1/52 B) 1/4 C) 1/2 D) 1/3

30. Jane rolled a die 7 times. Each time a 3 on the die showed up. What is the probability of Jane rolling a 3 on the next toss of the die?

    A) 1/6 B) 5/6 C) 3/6 DO 1/7

APPENDIX E

SAMPLE OF KAGAN'S MFF (Matching Familiar Figures) Test

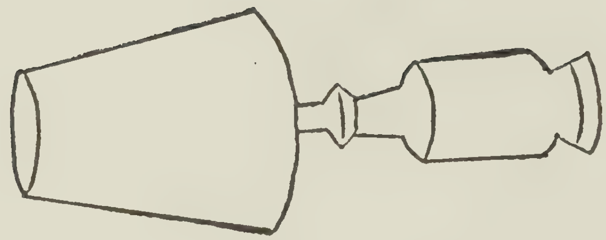
DIRECTIONS FOR MATCHING FAMILIAR PICTURES

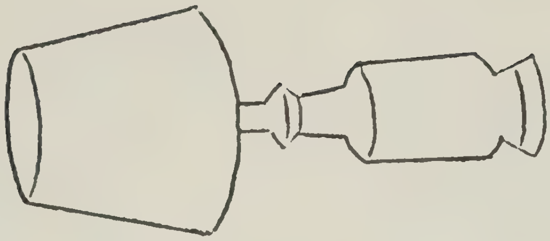
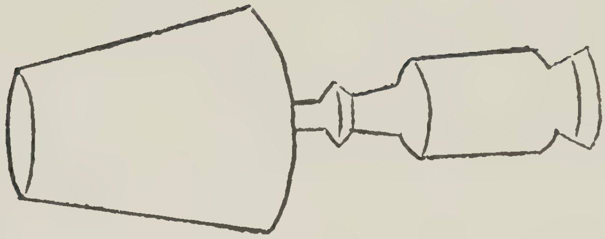
"I am going to show you a picture of something you know and then some pictures that look like it. You will have to point to the picture on this bottom page (point) that is just like the one on this top page (point). Let's do some for practice". E shows practice items and helps the child to find the correct answer. "Now we are going to do some that are a little bit harder. You will see a picture on top and eight pictures on the bottom. Find the one that is just like the one on top and point to it."

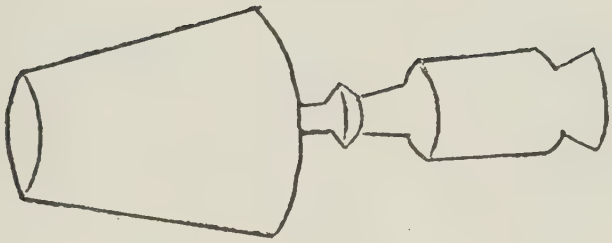
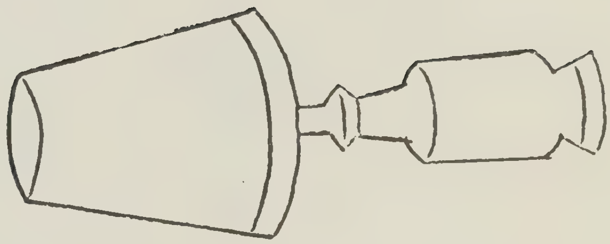
E will record latency to the first response to the half-second, total number of errors for each item and the order in which the errors are made. If S is correct, E will praise. If wrong, E will say, "No, that is not the right one. Find the one that is just like this one (point)." Continue to code responses (not times) until child makes a maximum of eight errors or gets the item correct. If incorrect E will show the right answer.

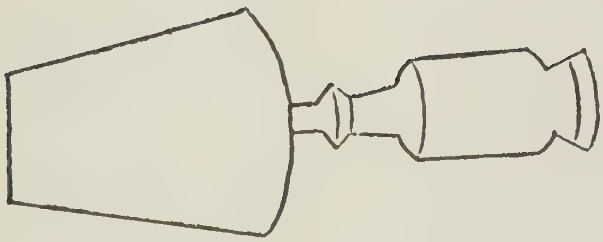
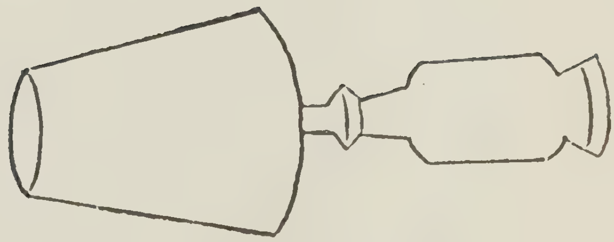
The test should be set up in a notebook. It is necessary to have a stand to place the book on so that both the stimulus and the alternatives are clearly visible to the S at the same time. The two pages should be practically at right angles to one another.

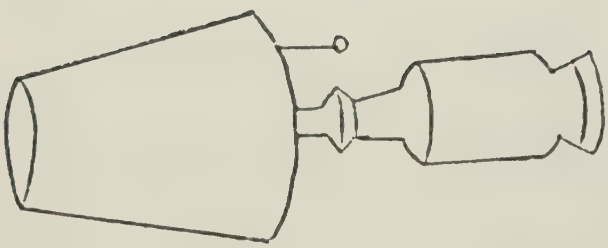
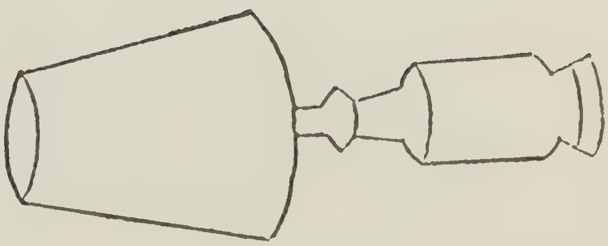
Note: It is desirable to insert the pages in clear plastic which helps to keep the pages clean.



APPENDIX F

SAMPLE OF SKEMP'S SK4 TEST

NAME .... SCHOOL ....
Last First Middle

AGE .... GRADE .... BOY GIRL (Circle One) DATE ....
Years Day Month Year

SK4: PART I

INSTRUCTIONS

There are 15 rows of figures in PART I of this test. In each row of figures, the first three figures (marked "EXAMPLES") all have some property in common. In the second group of three (marked "NOT EXAMPLES") in the row, none of the figures has this property. Your problem is to decide whether or not each of the figures under the question "ARE THESE EXAMPLES?" has the property. If it has the property (i.e., if it is an example), circle YES under that figure. If it does not have the property, circle NO.

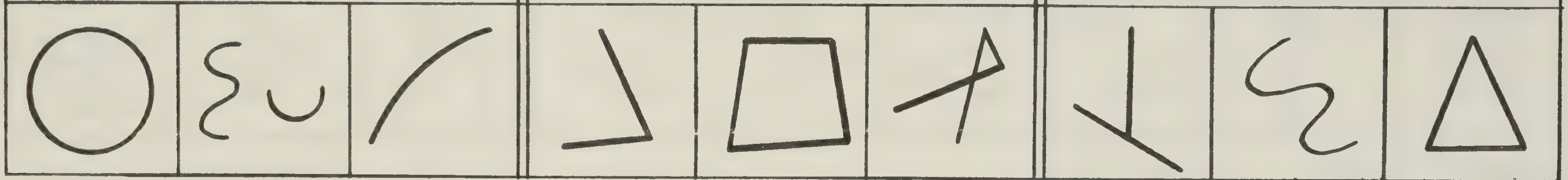
Look at the first row of figures at the top of the next page. Each of the figures under "EXAMPLES" is made of curved lines only. None of the figures under "NOT EXAMPLES" has this property. Look under the question "ARE THESE EXAMPLES?" Figure 1 is not made of curved lines, so you should circle NO for figure 1. Since figure 2 is curved, you should circle YES for figure 2. Figure 3 is not made of curved lines so you should circle NO for figure 3.

The second row of figures deals with another property. After deciding what the property is by looking at the figures under "EXAMPLES" and under "NOT EXAMPLES," answer the question "ARE THESE EXAMPLES?" by circling YES or NO for figure 4; then for figure 5; and then for figure 6. Then go on to the rest of the rows of figures in PART I of the test.

IMPORTANT: One, two, or all three of the figures under "ARE THESE EXAMPLES?" may have the property (i.e., may be examples).

Page 1

EXAMPLES	NOT EXAMPLES	ARE THESE EXAMPLES?

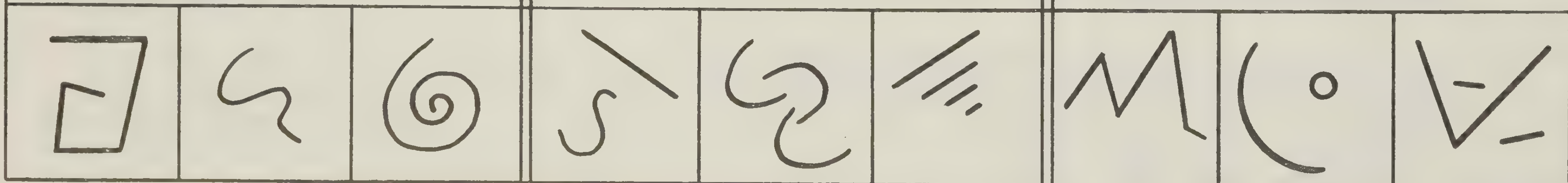


1. YES NO	2. YES NO	3. YES NO

EXAMPLES	NOT EXAMPLES	ARE THESE EXAMPLES?

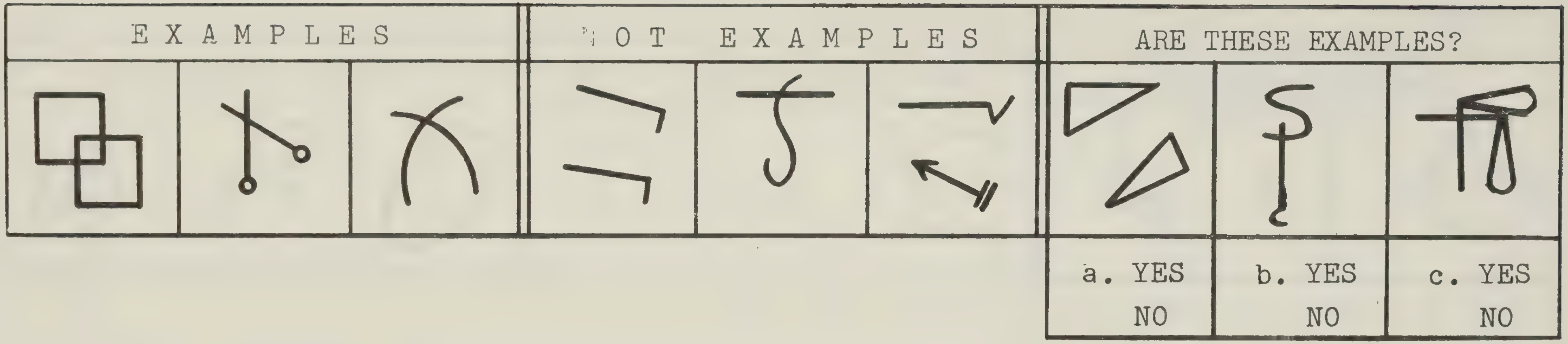
4. YES NO	5. YES NO	6. YES NO

EXAMPLES	NOT EXAMPLES	ARE THESE EXAMPLES?



7. YES NO	8. YES NO	9. YES NO

PRACTICE PROBLEM



INSTRUCTIONS

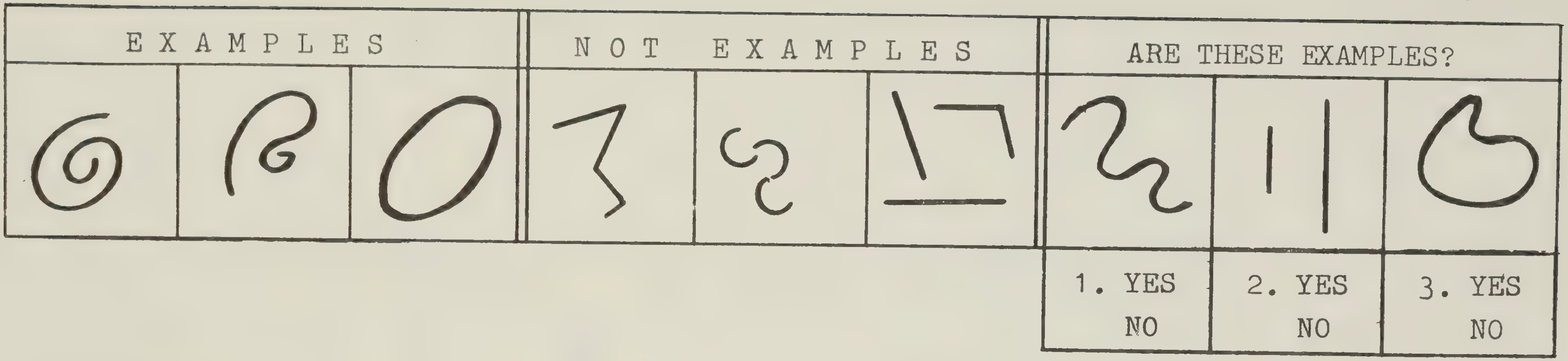
In each of the problems in PART II, the figures marked "EXAMPLES" have <u>two</u> properties in common. The figures marked "NOT EXAMPLES" do not have <u>both</u> of these properties. They may have only one, or neither. To help you, in each problem (row of figures) the first "NOT EXAMPLE" has one of the properties, the second has the other, and the third has neither.

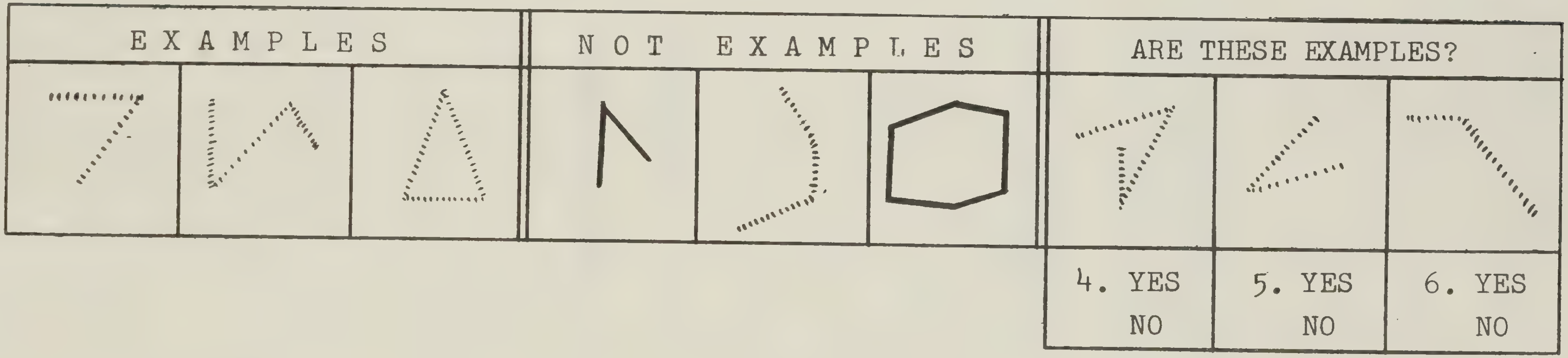
Your problem is to decide whether or not each of the figures under the question "ARE THESE EXAMPLES?" has both of the properties. If it has <u>both</u> properties, circle YES under that figure. If it does not have <u>both</u> properties, circle NO.

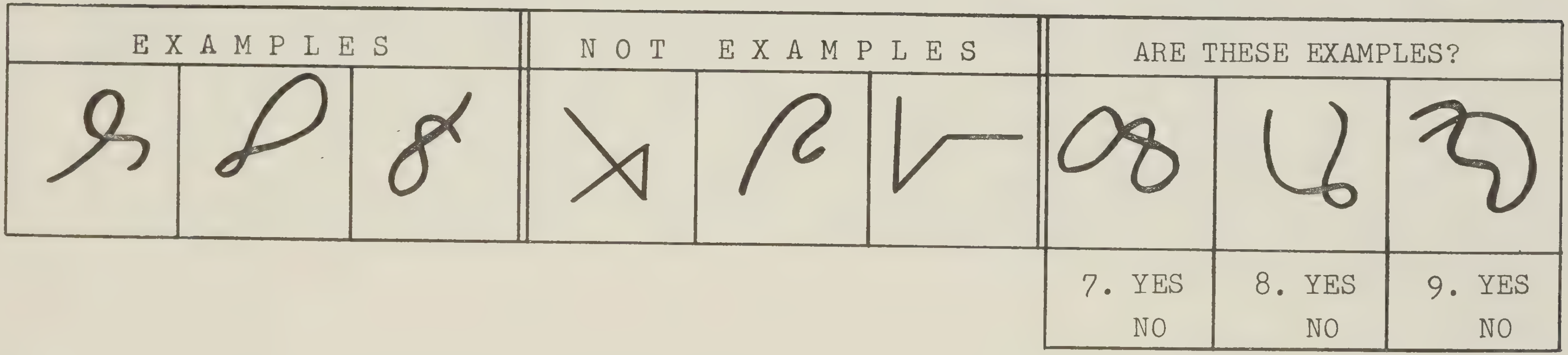
Look at the PRACTICE PROBLEM above. Each of the "EXAMPLES" is made up of two identical parts which intersect. In the first "NOT EXAMPLE," the parts are identical but do not intersect. In the second "NOT EXAMPLE," they intersect but are not identical. In the third "NOT EXAMPLE," they are neither identical nor intersecting.

For each of the figures under "ARE THESE EXAMPLES?", decide whether or not it has <u>both</u> properties. In the PRACTICE PROBLEM, figure a. has two identical parts but they do not intersect, so you should circle NO. Figure b. does have two intersecting parts but they are not identical, so circle NO under figure b. Figure c. does have two identical parts <u>and</u> they do intersect so circle YES under figure c.

All of the problems in PART II are of the same kind as the PRACTICE PROBLEM, BUT one, two, or all three of the figures under "ARE THESE EXAMPLES?" may be examples.


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
1. YES NO
2. YES NO
3. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
4. YES NO
5. YES NO
6. YES NO


EXAMPLES
NOT EXAMPLES
ARE THESE EXAMPLES?
7. YES NO
8. YES NO
9. YES NO

APPENDIX G

SAMPLE OF SKEMP'S SK6 TEST

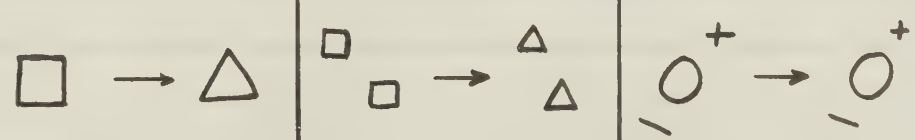
Operation 1	

In the above figures, the one on the left of each pair has been changed to the one on the right by means of the same simple operation. In other words, the above figures give three examples of a particular operation. You have to find out what the operation is, and then do the same operation to some other figures.

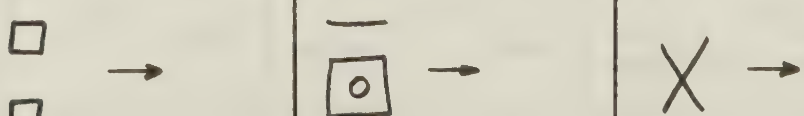
What is the operation? It is reversing from left to right. Do this on each of the figures below, and fill in the answers in the blank spaces. Check with the answers on the blackboard to make sure that you have understood.

Do Operation 1 on these.	

Here is a different operation:

Operation 2	

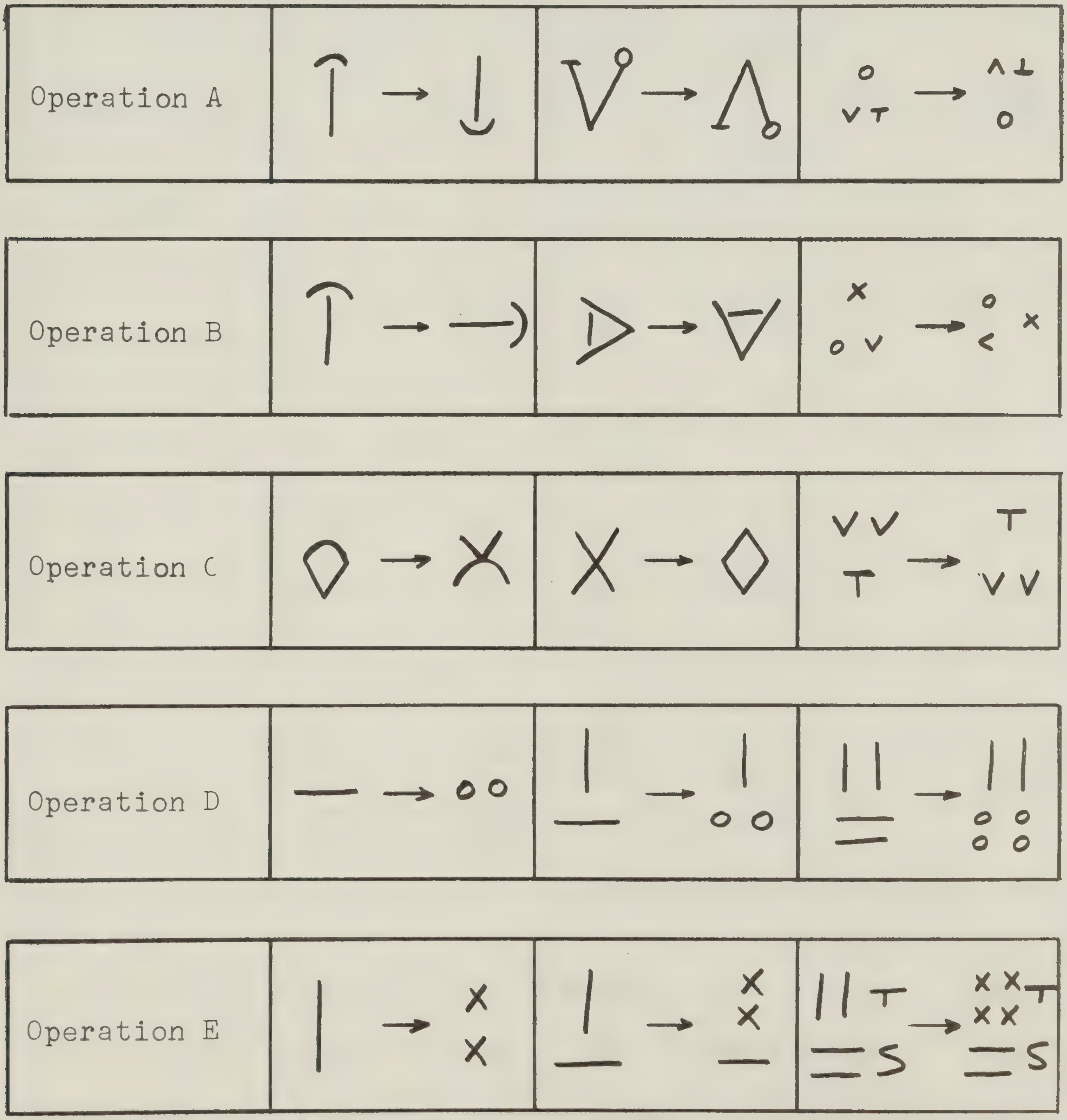
When you have found out what it is, do it on the figures below. Check with the answers on the board.

Do Operation 2 on these.	

SK6: DEMONSTRATION SHEET

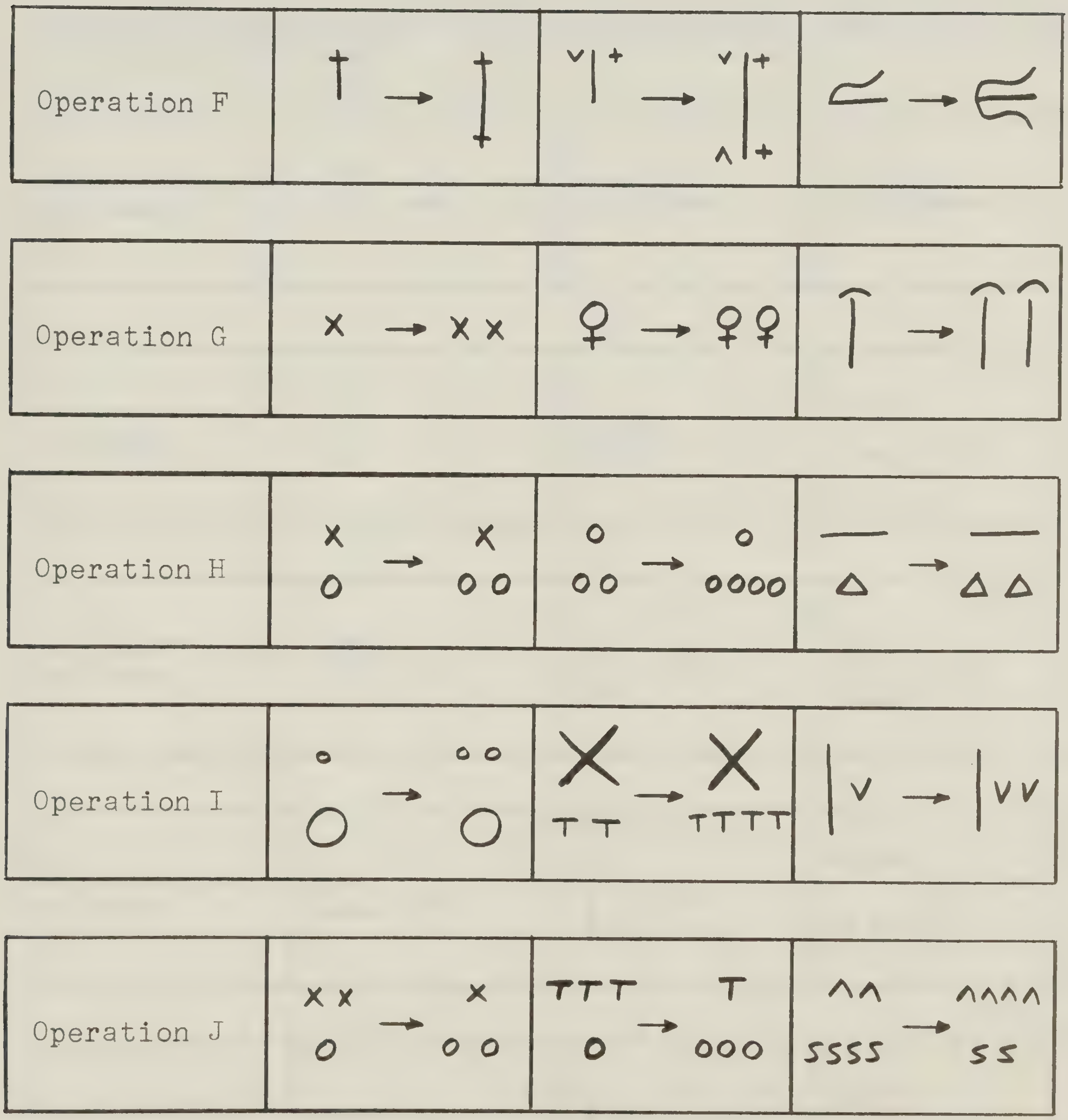
OPERATIONS A TO E

(OPERATIONS F TO J ARE ON THE NEXT PAGE)



SK6: DEMONSTRATION SHEET

OPERATIONS F TO J

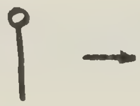
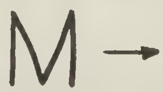
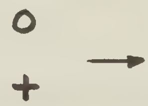
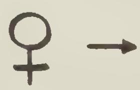
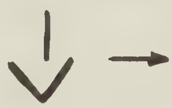
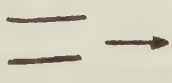
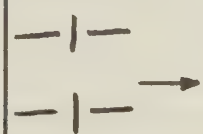
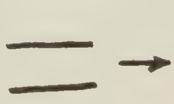
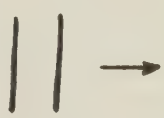
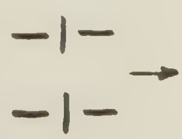


NAME .... SCHOOL ....
Last First Middle

AGE .... GRADE .... BOY GIRL DATE ....
Years (Circle One) Day Month Yr.

SK6: PART I

Find out the operations from the DEMONSTRATION SHEET, and fill in the answers in the blank spaces, just as you did on the PRACTICE SHEET.

Do Operation A on these.			
Do Operation B on these.			
Do Operation C on these.			
Do Operation D on these.			
Do Operation E on these.			

NAME .... SCHOOL ....
Last First Middle

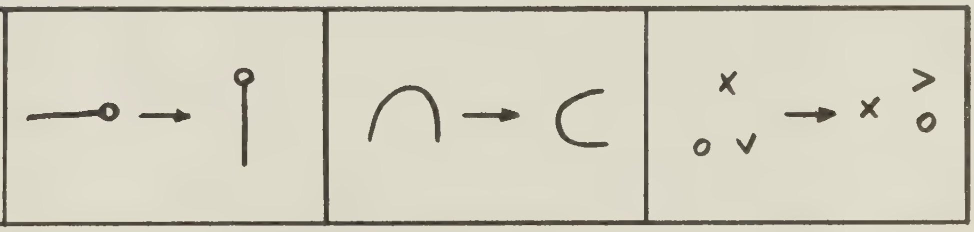
AGE .... GRADE .... BOY GIRL DATE ....
Years (Circle One) Day Month Yr.

SK6: PART II

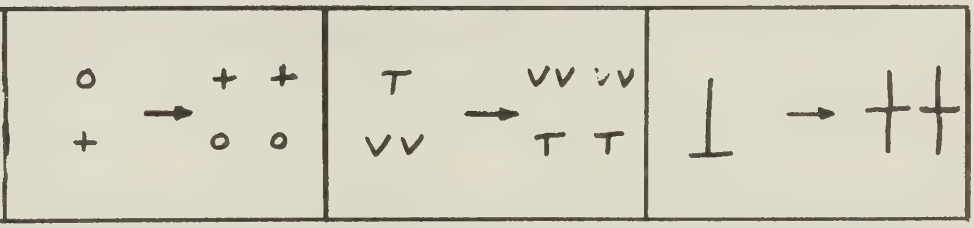
In PART II the problem is to combine the operations on the DEMONSTRATION SHEET, or to do them in reverse, or both. When combining operations, they are to be done in the order given (i.e., "Combine C and G" means "Do Operation C first and then do Operation G.")

Look at the examples given below and then carry out the operations indicated on the following three pages.

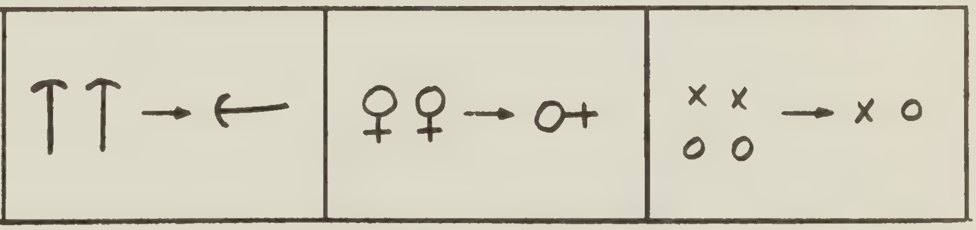
EXAMPLE:
Reverse B



EXAMPLE:
Combine C & G



EXAMPLE:
Reverse and
Combine G & B



APPENDIX H

RAW DATA COLLECTED IN THE STUDY

Student No.	Attitude	Skemp	Kagan Error	Kagan Total Time	Achievement	Verbal I.Q.	Non-Verbal I.Q.	Directed Lab=1 Otherwise Non-Directed Lab	Reflective = 1	Impulse = 1	Unclassified = 1
1	83	13	6	757.5	16	100	86	1	1	0	0
2	34	10	4	412.0	15	99	107	1	1	0	0
3	108	20	6	1153.5	14	110	110	1	1	0	0
4	96	6	10	406.5	10	113	112	1	1	0	0
5	94	24	2	784.0	21	104	105	1	1	0	0
6	87	10	6	687.0	15	115	107	1	1	0	0
7	56	20	10	410.0	15	117	108	1	1	0	0
8	96	31	1	969.5	18	116	122	1	1	0	0
9	74	2	10	830.0	12	101	86	1	1	0	0
10	77	4	8	530.5	12	101	102	1	1	0	0
11	113	7	0	702.0	10	110	98	1	1	0	0
12	78	7	6	672.0	10	104	101	1	1	0	0
13	85	22	3	753.0	13	85	117	1	1	0	0
14	85	26	7	511.0	23	120	120	1	1	0	0
15	95	6	8	460.5	19	116	125	1	1	0	0
16	108	14	5	659.5	17	122	102	0	1	0	0
17	78	33	0	1268.5	25	111	128	0	1	0	0
18	63	12	1	1082.0	13	115	117	0	1	0	0
19	75	6	3	733.0	21	110	10[illegible]	0	1	0	0
20	[illegible]	17	11	570.5	10	130	131	0	1	0	0
21	106	17	11	3[illegible].5	21	111	114	0	1	0	0

Student No.	Attitude	Skemp	Kagan Error	Kagan Total Time	Achievement	Verbal I.Q.	Non-Verbal I.Q.	Directed Lab=1 Otherwise Non-Directed Lab	Reflective = 1	Impulse = 1	Unclassified = 1
22	71	7	5	470.5	20	120	124	0	1	0	0
23	110	9	8	2016.5	12	120	101	0	1	0	0
24	93	2	4	1452.5	8	96	112	0	1	0	0
25	61	10	3	1562.5	11	122	114	0	1	0	0
26	75	17	10	414.0	21	99	102	0	1	0	0
27	101	14	31	134.0	21	117	110	1	0	1	0
28	35	10	17	361.5	9	113	104	1	0	1	0
29	109	24	20	134.5	15	130	117	1	0	1	0
30	102	22	17	159.5	18	92	126	1	0	1	0
31	108	13	22	259.0	15	95	101	1	0	1	0
32	83	20	22	323.5	13	86	96	1	0	1	0
33	68	4	19	197.5	13	113	120	1	0	1	0
34	100	8	23	149.0	17	115	117	1	0	1	0
35	103	32	13	253.5	13	95	105	0	0	1	0
36	93	17	27	177.0	13	110	99	0	0	1	0
37	100	16	21	393.0	14	111	131	0	0	1	0
38	88	5	20	223.0	14	97	110	0	0	1	0
39	100	12	14	536.5	15	110	98	0	0	1	0
40	63	13	18	359.0	16	103	122	0	0	1	0
41	33	4	23	55.0	13	100	110	0	0	1	0
42	79	14	21	240.0	11	103	119	0	0	1	0

Student No.	Attitude	Skemp	Kagan Error	Kagan Total Time	Achievement	Verbal I.Q.	Non-Verbal I.Q.	Directed Lab=1 Otherwise Non-Directed Lab	Reflective = 1	Impulse = 1	Unclassified = 1
43	39	24	15	347.5	16	95	102	0	0	1	0
44	67	9	21	298.5	15	101	98	0	0	1	0
45	95	22	13	303.0	17	115	105	0	0	1	0
46	75	2	16	261.5	15	108	96	0	0	1	0
47	101	23	7	369.5	16	87	130	1	0	0	1
48	83	26	14	449.5	13	110	118	1	0	0	1
49	103	25	7	325.0	13	104	111	1	0	0	1
50	87	13	10	348.0	13	134	133	1	0	0	1
51	108	21	12	195.5	17	100	110	1	0	0	1
52	80	23	12	296.5	23	115	102	1	0	0	1
53	63	26	13	420.5	19	100	125	1	0	0	1
54	98	7	15	584.0	19	101	106	1	0	0	1
55	57	11	12	334.0	7	93	116	1	0	0	1
56	120	21	12	370.0	14	115	109	1	0	0	1
57	68	27	19	400.5	20	110	108	1	0	0	1
58	101	6	12	327.5	17	112	104	1	0	0	1
59	93	0	13	510.0	10	108	124	0	0	0	1
60	94	10	10	202.5	21	116	120	0	0	0	1
61	80	14	12	370.0	9	91	99	0	0	0	1
62	86	4	12	293.5	14	106	98	0	0	0	1
63	76	0	13	507.5	13	121	108	0	0	0	1

APPENDIX I

DIENES' THINKING STRUCTURES -

OPERATIONAL, PATTERN AND MEMORY THINKING

Diene's Operational, Pattern and Memory Thinking

Background -

"Thinking has only relatively recently been studied upon an experimental basis, if by thinking is meant the progression as a result of direction by the subject through a sequence of related stages to one which is regarded by the subject as a natural or satisfying end." (Dienes, 1965, p. 1)

Bruner and his co-workers have broken thinking down into components. The problem of working experimentally on part of the thinking process is that thinking is not equal to the sum of its parts. Dienes attempted to overcome this problem by discussing thinking in structures where structures are defined as the way in which parts make up the whole. A structure is a set of relationships and/or interdependencies between events. The number of independent variables gives the dimension of the structure.

"If an event is entirely determined by giving the value of a single variable the structure is one dimensional. In some one-dimensional structures an event is determined by the immediately preceding event, in others by the set of all the events preceding the event being considered. Recurring decimals would be examples of the former, non-recurring decimals of the latter possibility. In the former the number of values that the one variable may assume is finite ...
, in the latter the variable may assume an infinite number of finite sequences that can be built out of the ten digits." (Dienes, 1965, p. 17).

Most events in life have many variables affecting the outcome. Models are used to predict the maximum number of correct answers. If the model enables one to predict accurately at that moment in time, then the model is a good one. Dienes, for his experimental model, used a two-dimensional structure to predict what individual strategies in thinking naturally subdivided into distinguishable groups. He found three main groups -- operational, pattern and memory. The memory strategy is an individual's learning strategy which is random since each solution is

memorized independently of the others. This is the lowest level in the hierarchy of thinking with respect to mathematics. Next is the pattern strategy where an individual searches for a general pattern or table on which to place solutions and finally, the operational strategy is one where the individual presupposes that part of the pattern acts on some other part. This implies that part of the pattern is a function of another part.

Instrument -

The apparatus consisted of a piece of hardboard with a window cut into it and two identical sets of cards. The subject being tested was given the following instructions:

"We are going to play a game with you, and your task will be to discover the rules of the game as we play it. You have these two (four) cards. I also have two (four) such cards, with the same pictures (or signs), which I can put in this window.

What I shall want you to do is:
(1) Look in the window, then
(2) Play one of your cards, by putting one on the table so that I can see it.

Then I shall shut the window, then open it again with another picture or the same picture showing. What you will see in the window next will depend entirely on what was in the window before and on the card you played ..." (Dienes, 1965, P. 21).

Dienes used a two-dimensional structure based on the mathematical group to test for different thinking structures. The two independent variables were the cards which were placed in the window and the cards that were played by the subject being tested. There were either two or four values which the cards could have.

"The mathematical group with two elements has the following rule structure:

Let (a) and (b) denote the elements. Then
(a) with (a) will yield (a)
(a) with (b) will yield (b)
(b) with (a) will yield (b)
(b) with (b) will yield (a)." (Dienes 1965, P. 17).

Theoretical Framework -

Dienes found three distinct groups of thinking -- operational, pattern and memory. These groups are hierarchical in nature with the operational strategy being the most powerful method from the mathematical point of view. It was shown that adults tend to use operational strategies more often than children. With these arguments in mind, it would seem reasonable to hypothesize that operational thinkers should do better than pattern thinkers and the latter better than memory thinkers on an achievement test of the work done in the experimental laboratory setting.